An approximate Tutte-decomposition for arbitrary connectivity

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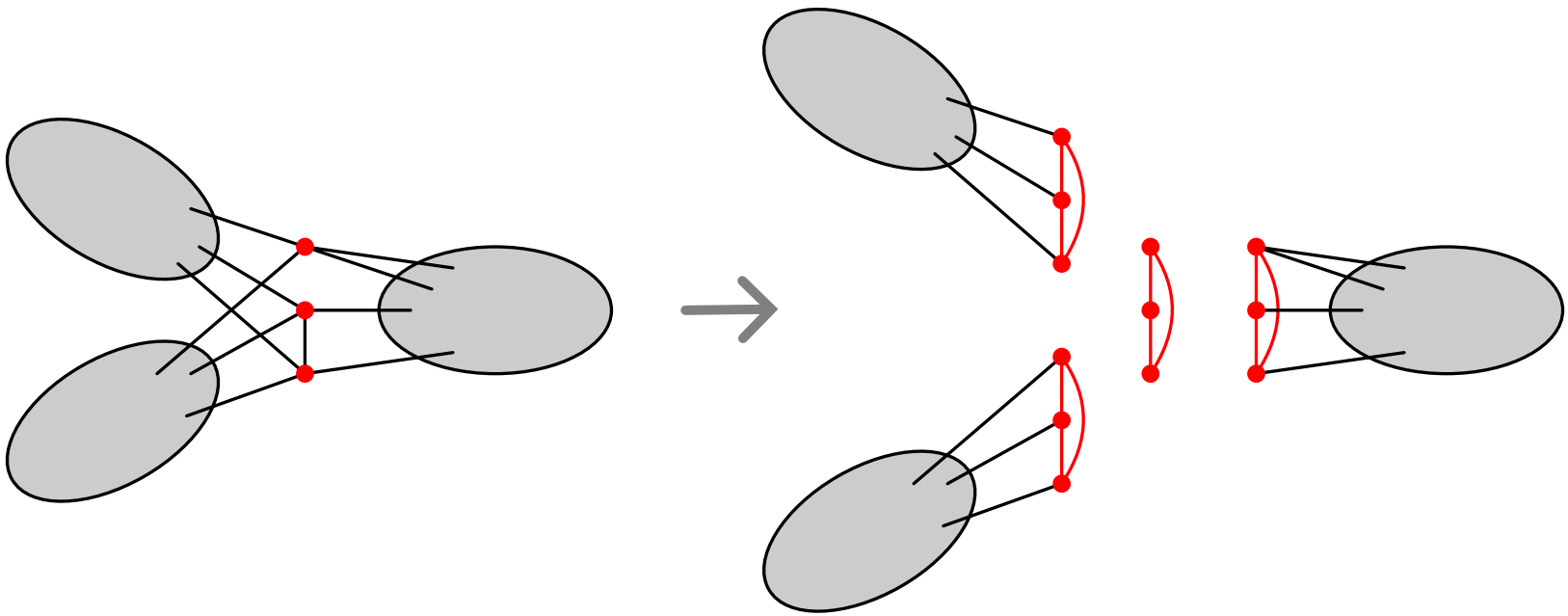
Joint work with Romain Bourneuf, Johannes Carmesin and Tim Planken

Leipzig '26

Problem: Canonically decompose k-con'd G along k-separators into parts that are $(k+1)$-con'd or 'basic'.

canonical $:\Longleftrightarrow$ Aut(G)-invariant

Decomposing G along a k-separator:



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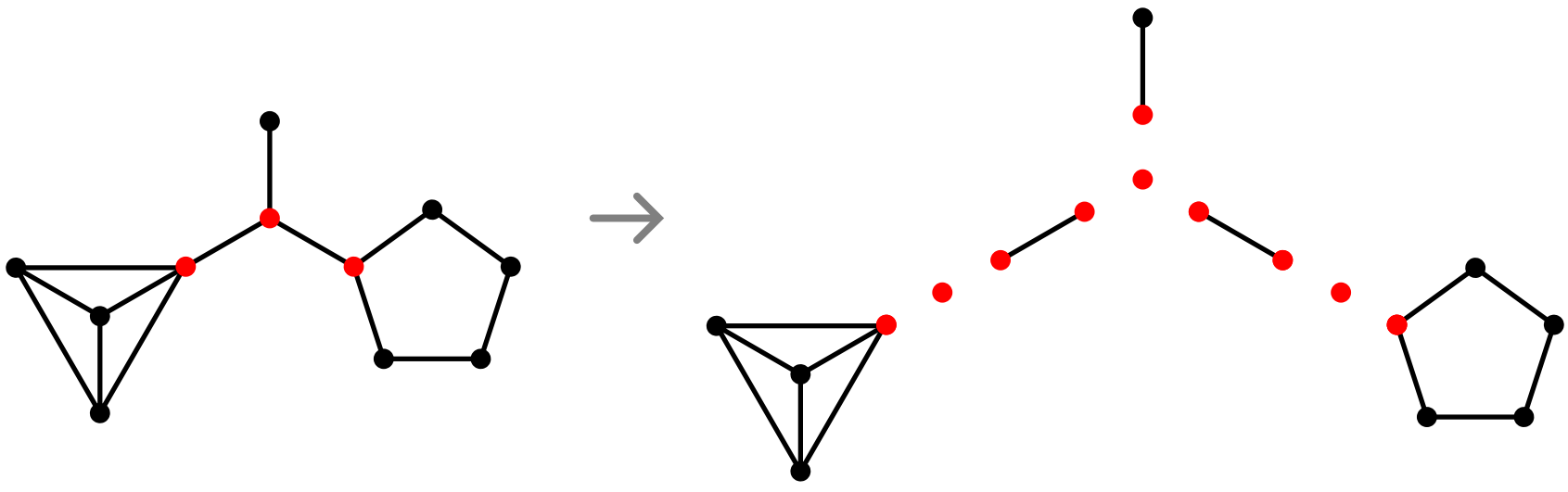

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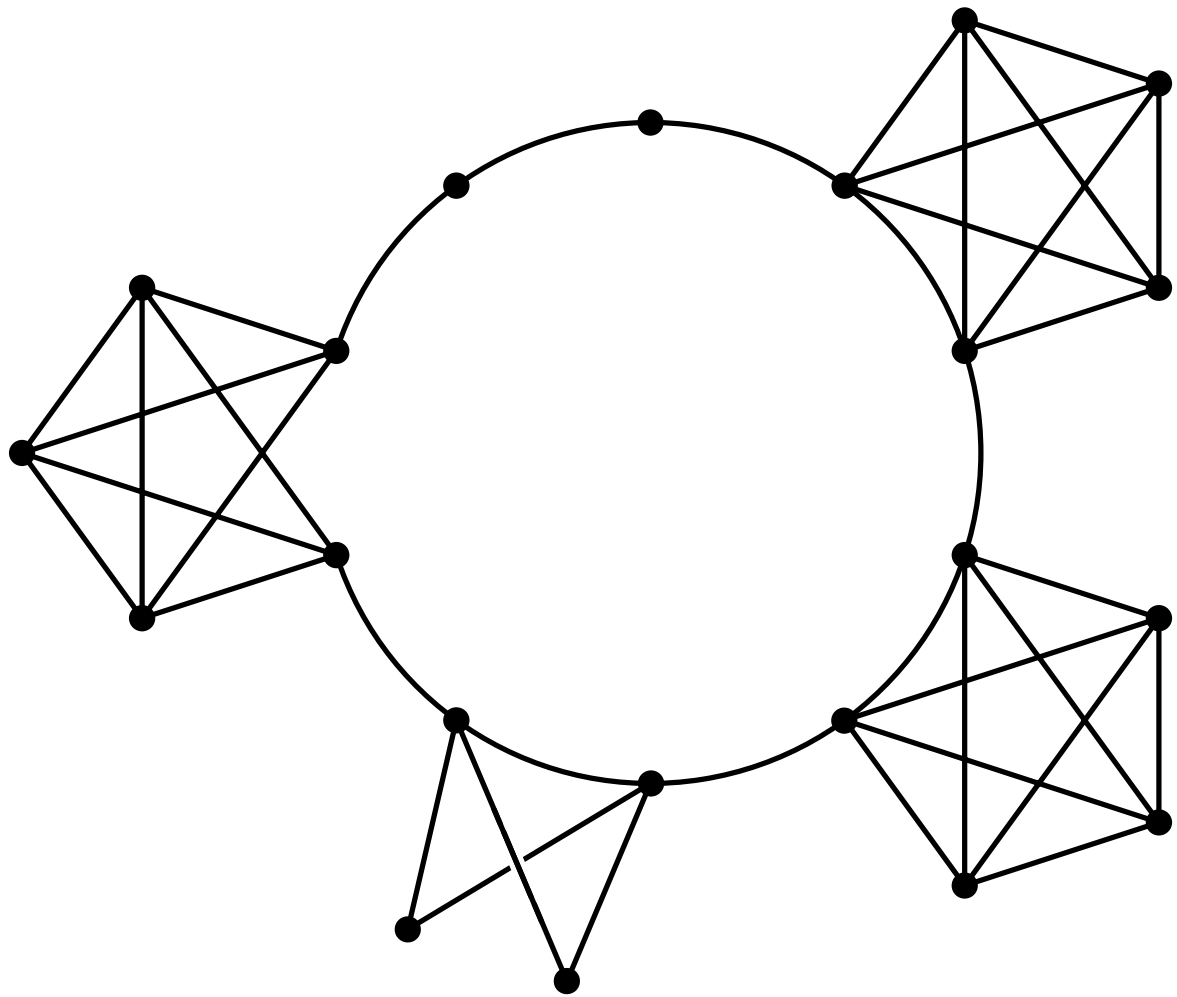

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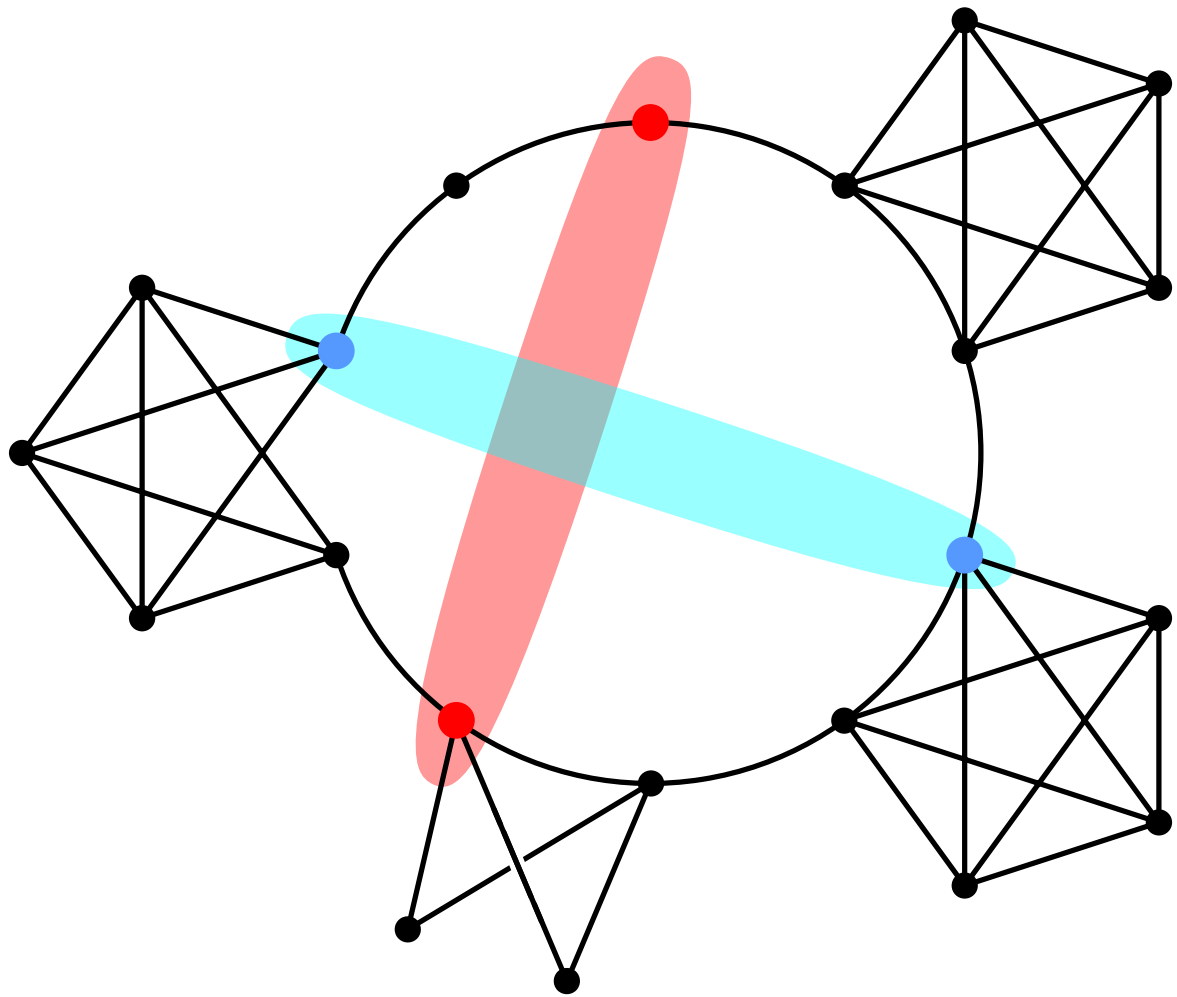

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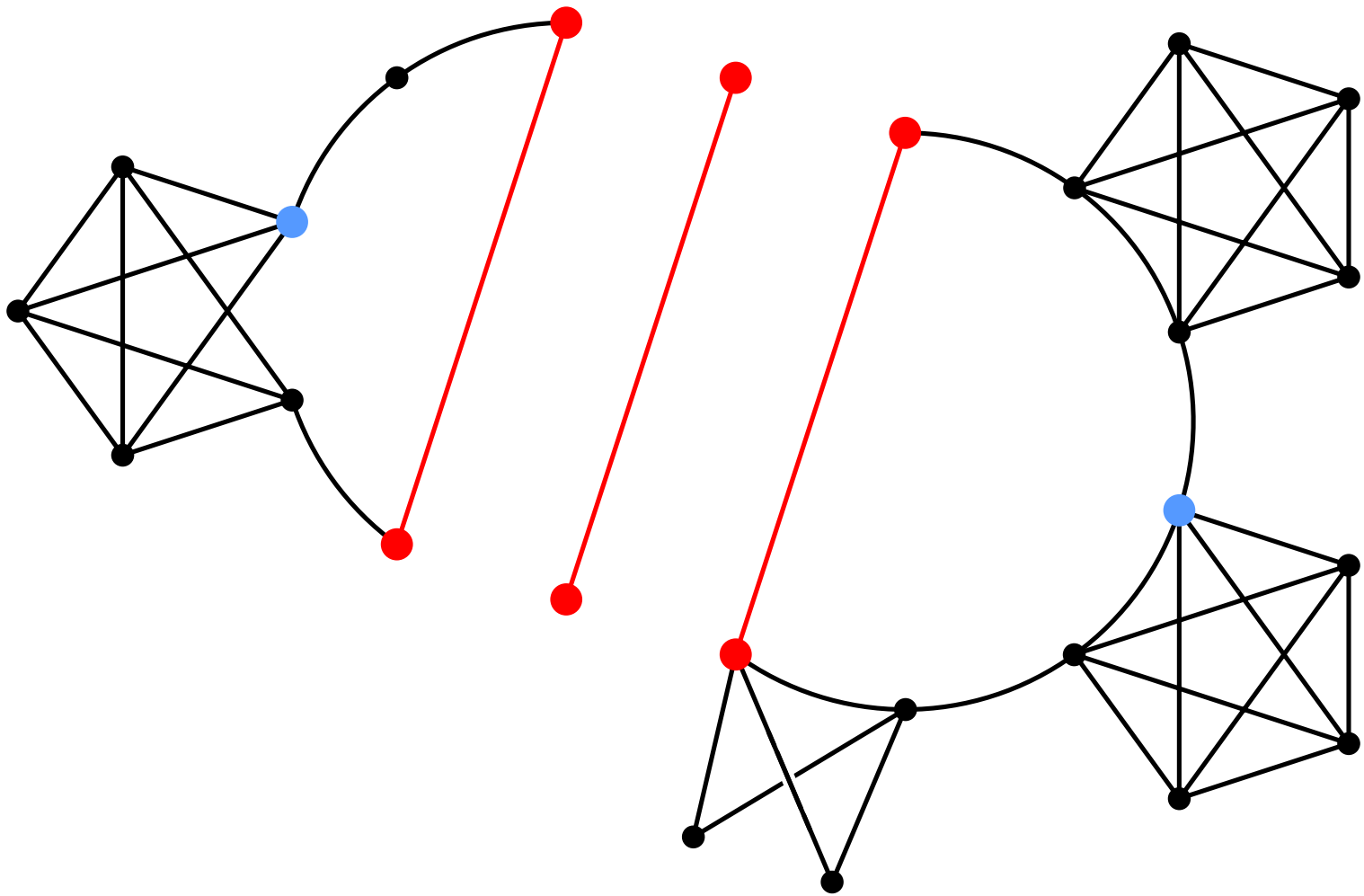
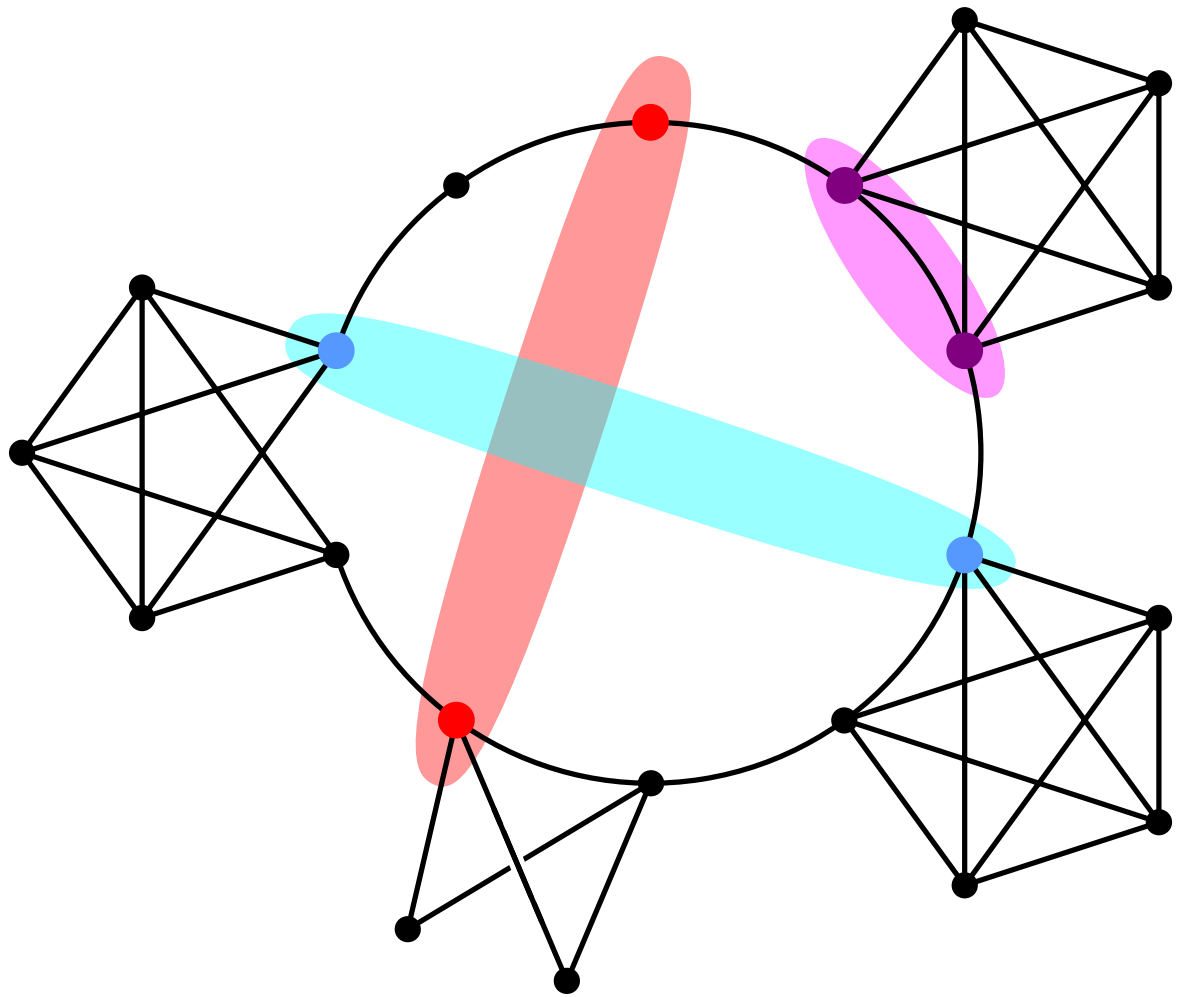


Two k-separators *cross* if they separate each other;

otherwise they are *nested*.



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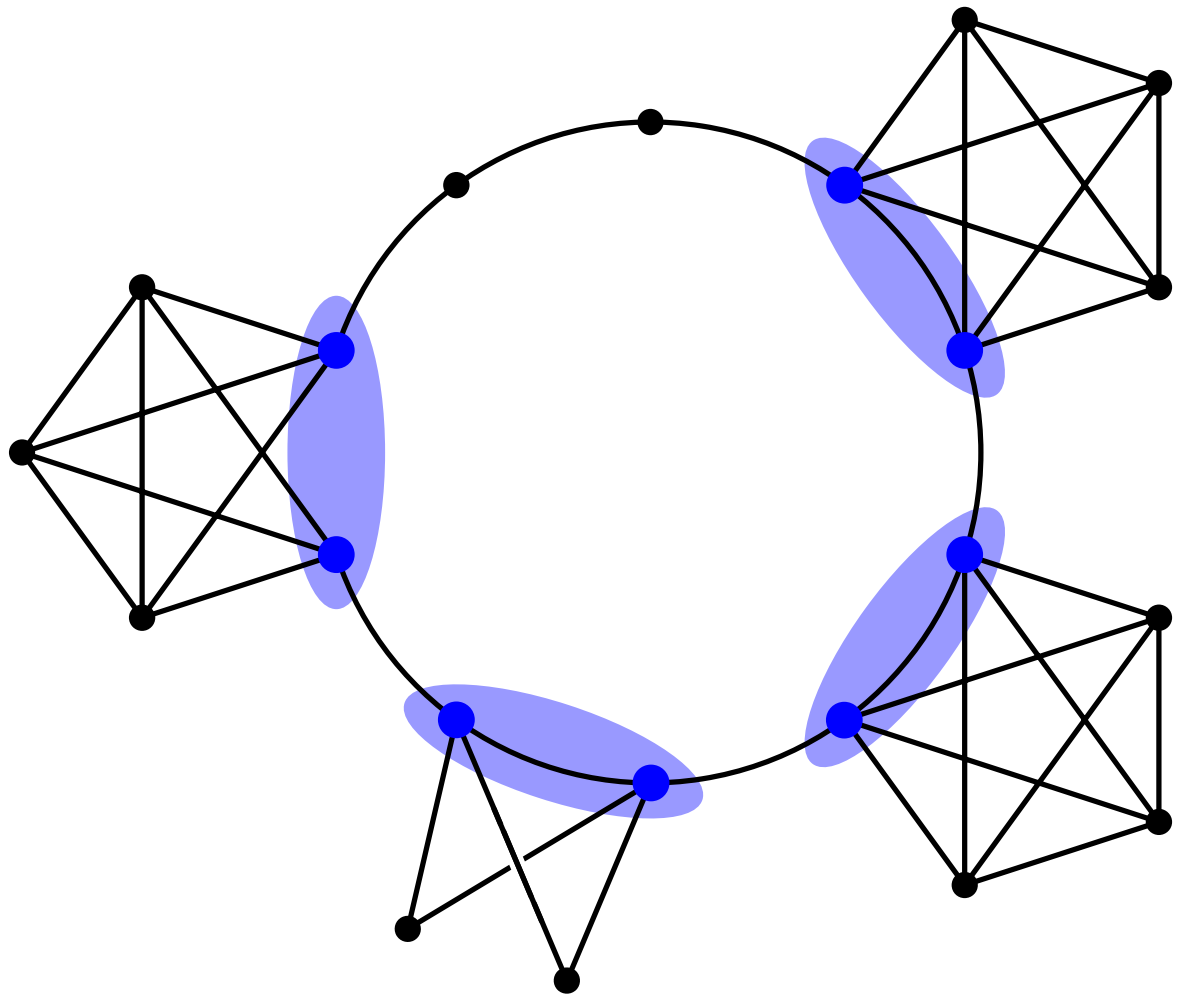

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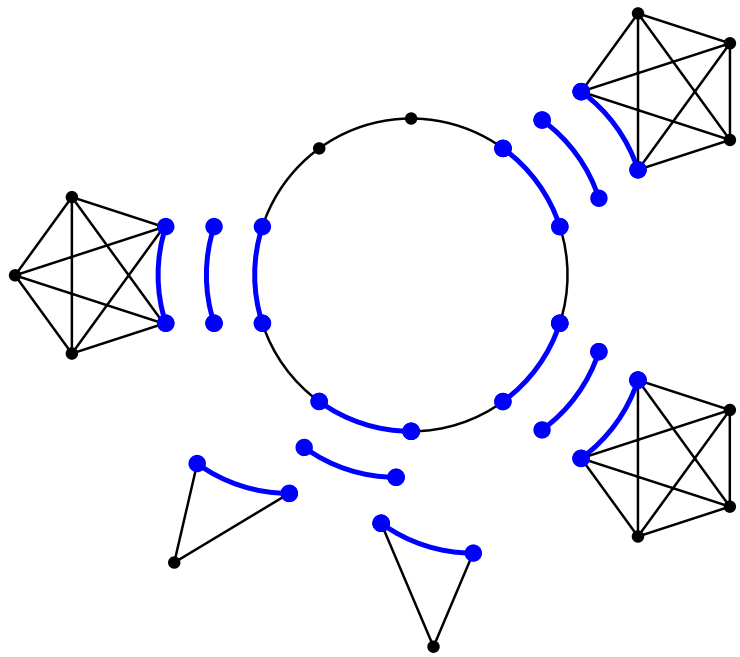

Theorem (Tutte 66)

Every 2-con'd G decomposes along its totally-nested 2-separators into 3-con'd graphs, cycles and K_2's.

Guess: Every k-con'd G decomposes along its totally-nested k-separators into $(k+1)$-con'd graphs and 'basic' graphs.

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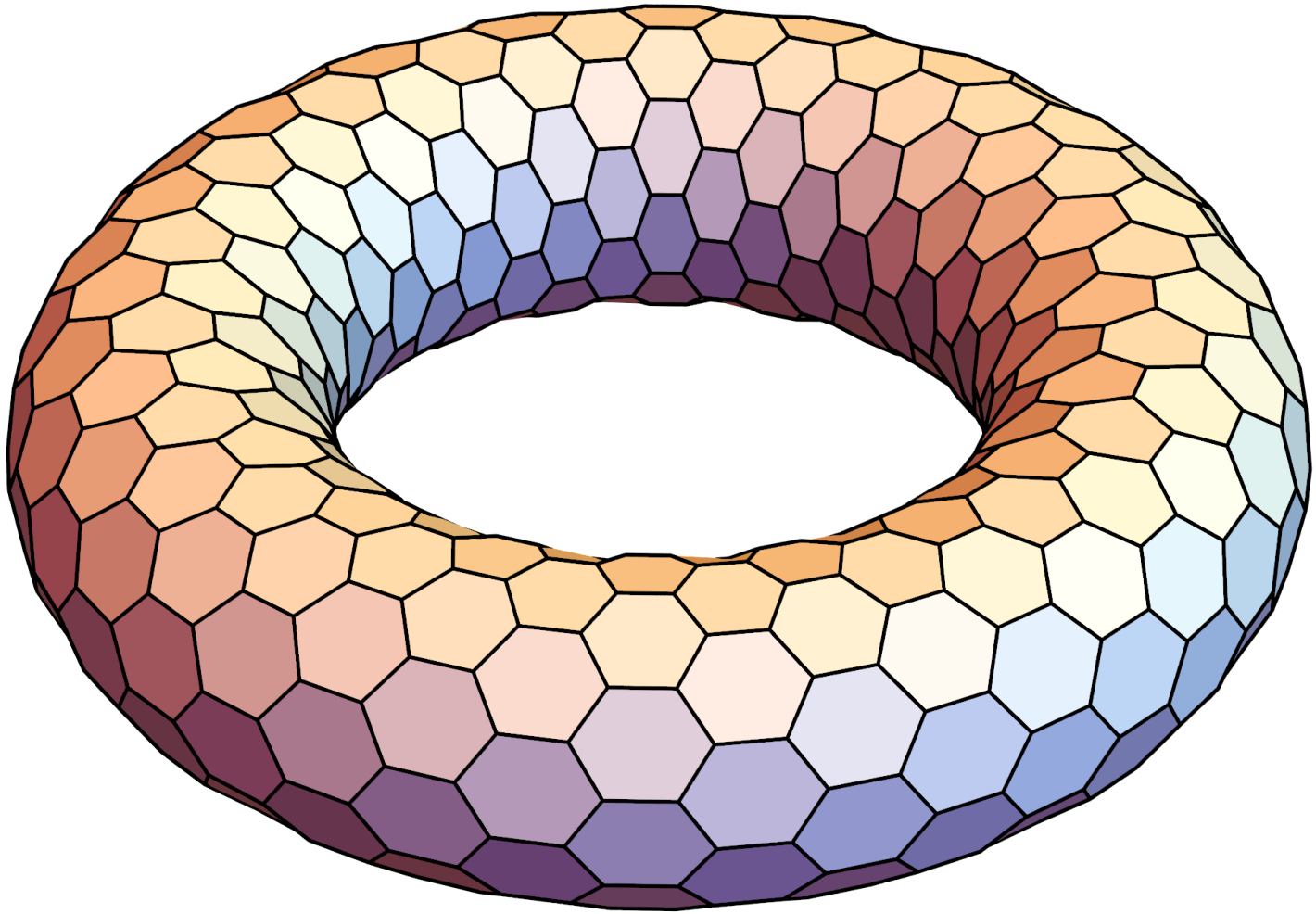
Challenge 1 (Figure: $k = 3$)



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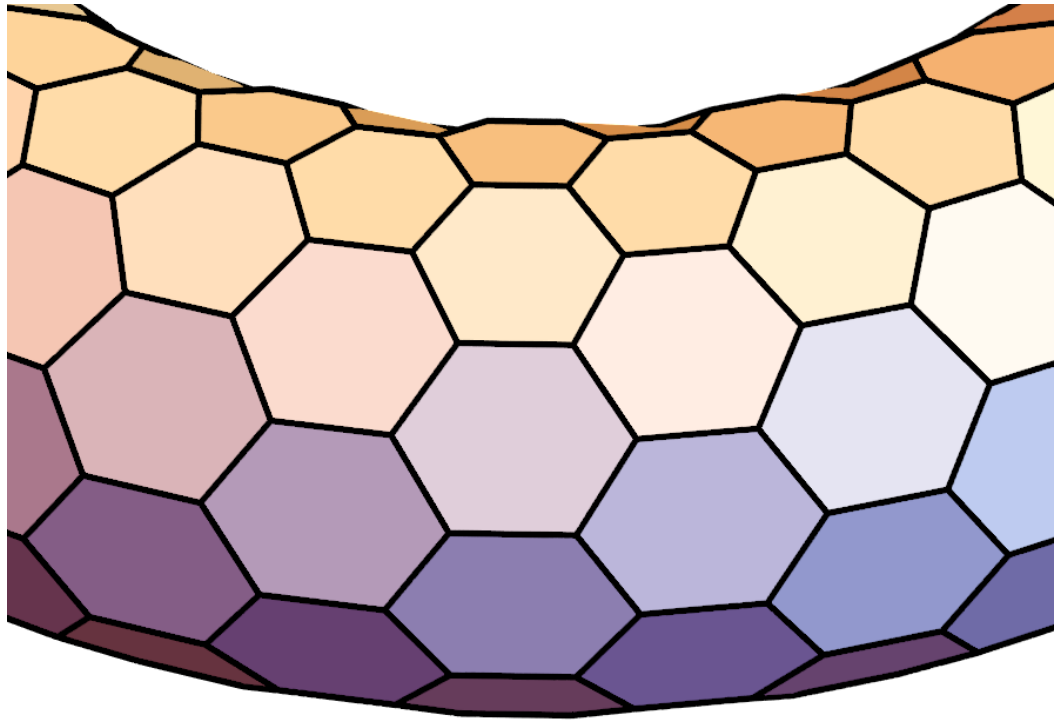

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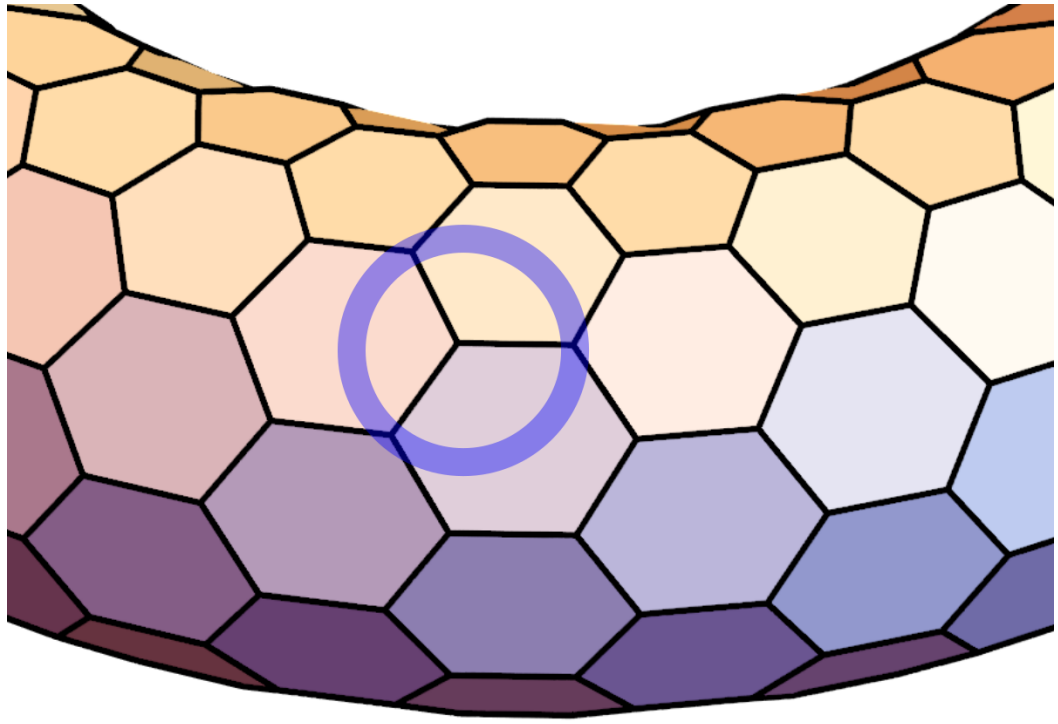

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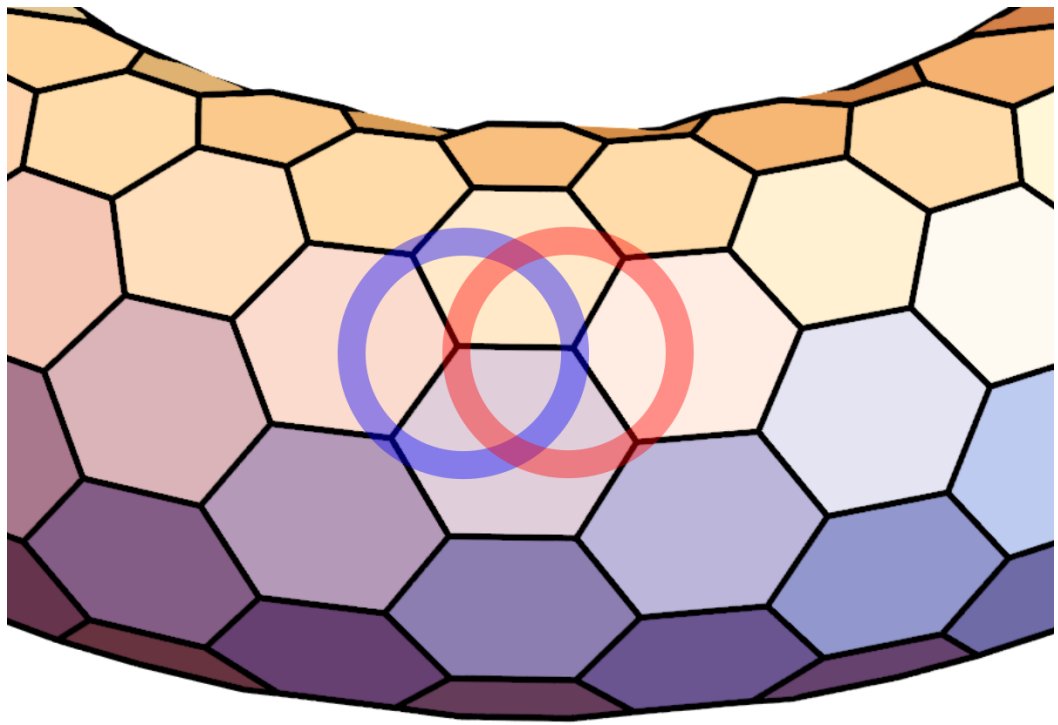

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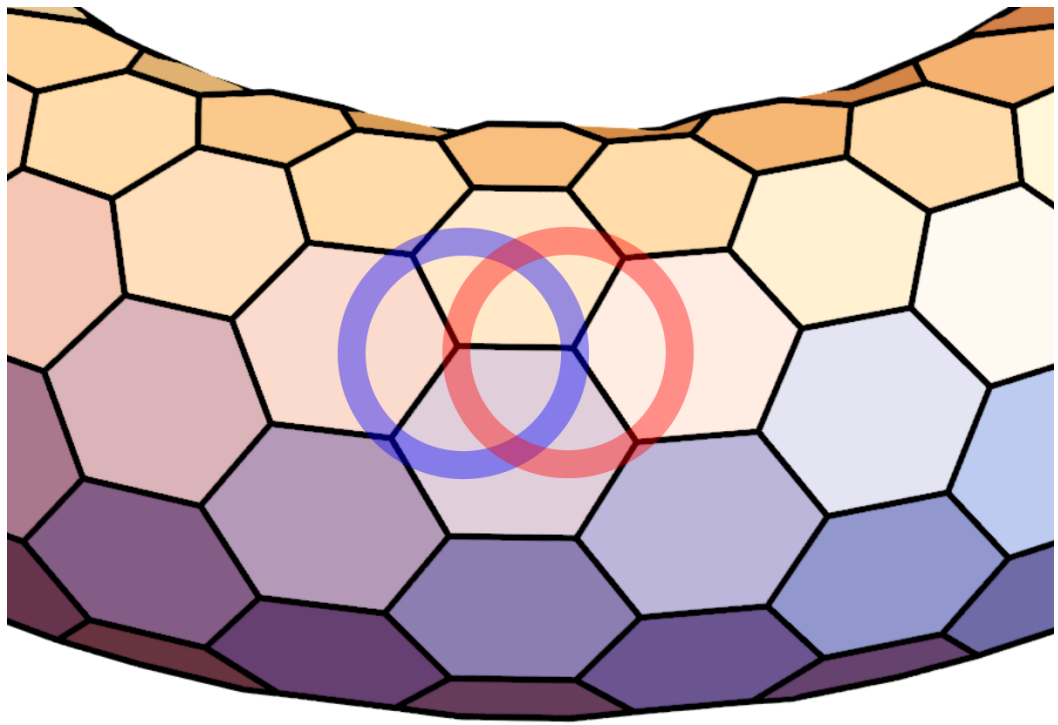

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Guess: Every k-con'd G decomposes along its totally-nested k-separators into *quasi-$(k+1)$-con'd* graphs and 'basic' graphs.

$:\Longleftrightarrow$ k-con'd and every k-sep'r cuts off only one vertex

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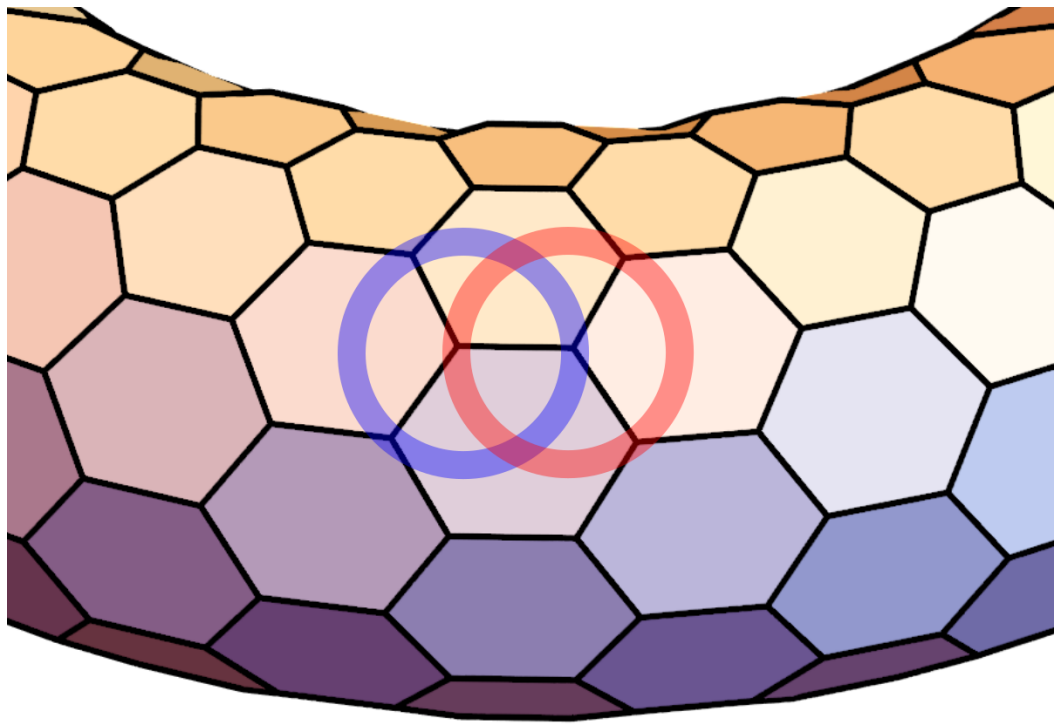

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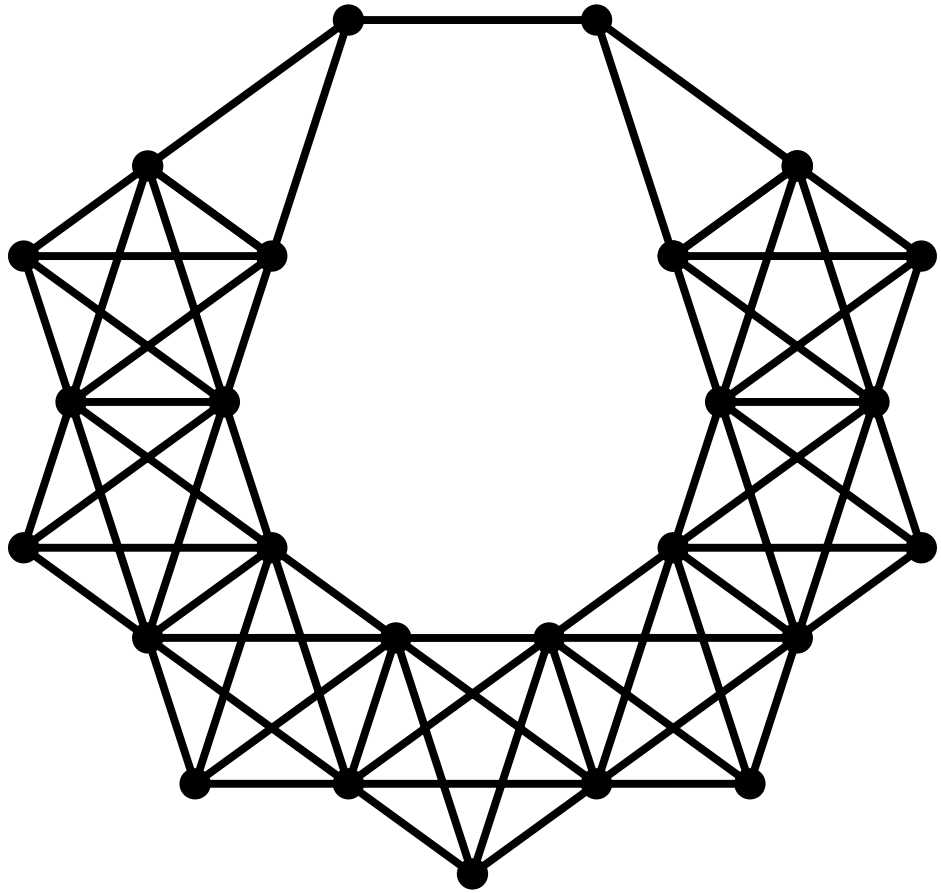

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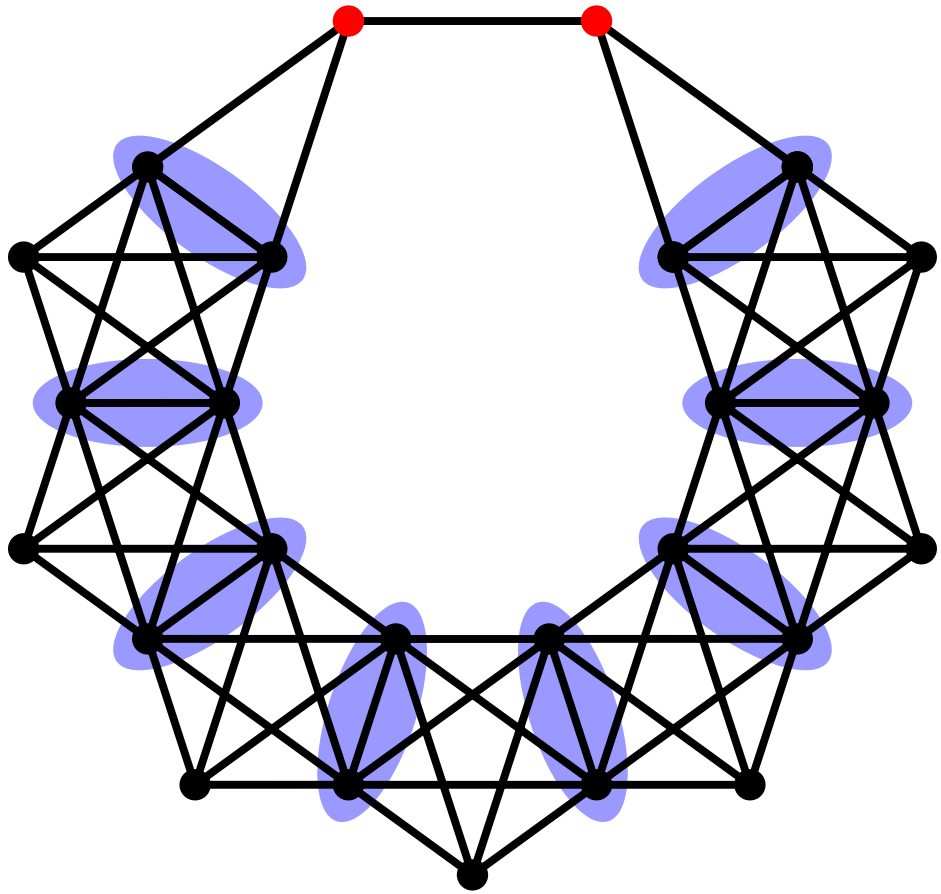

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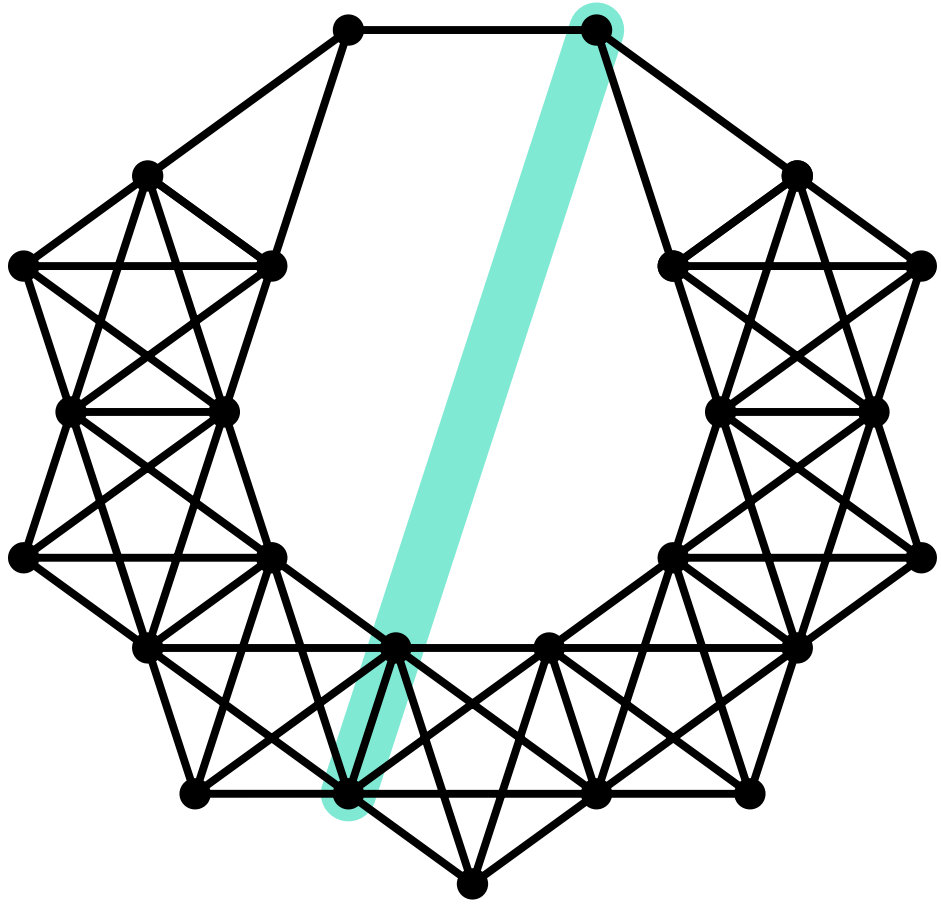

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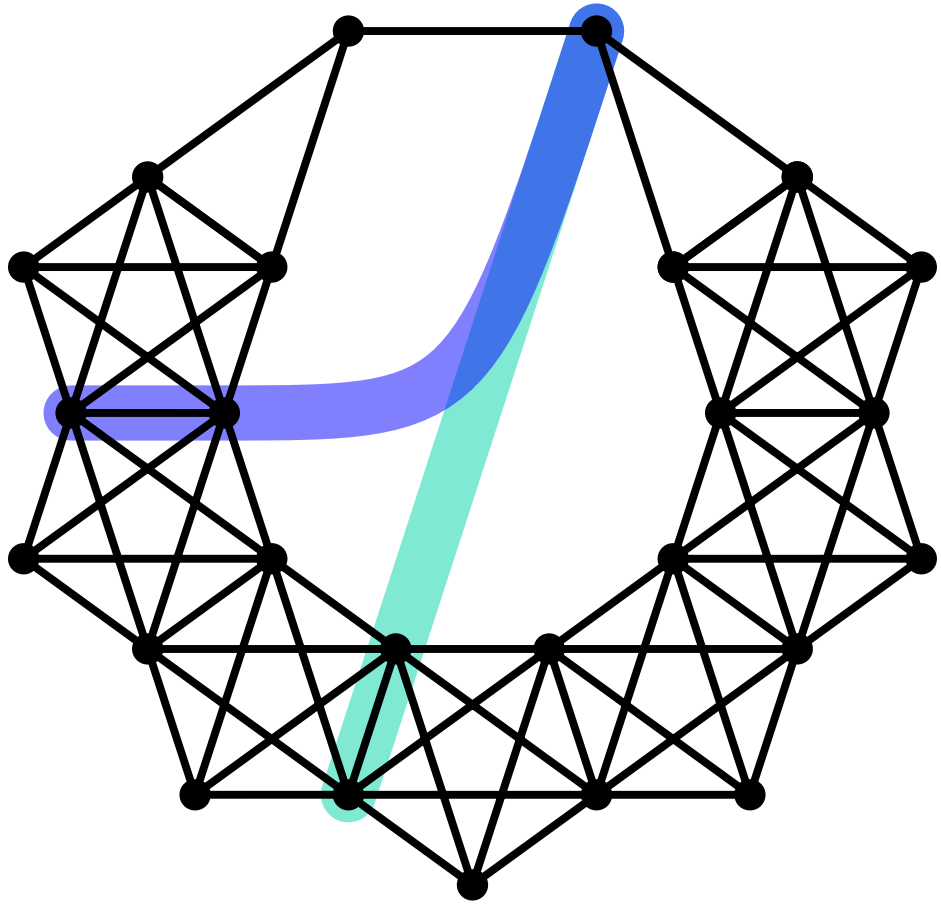

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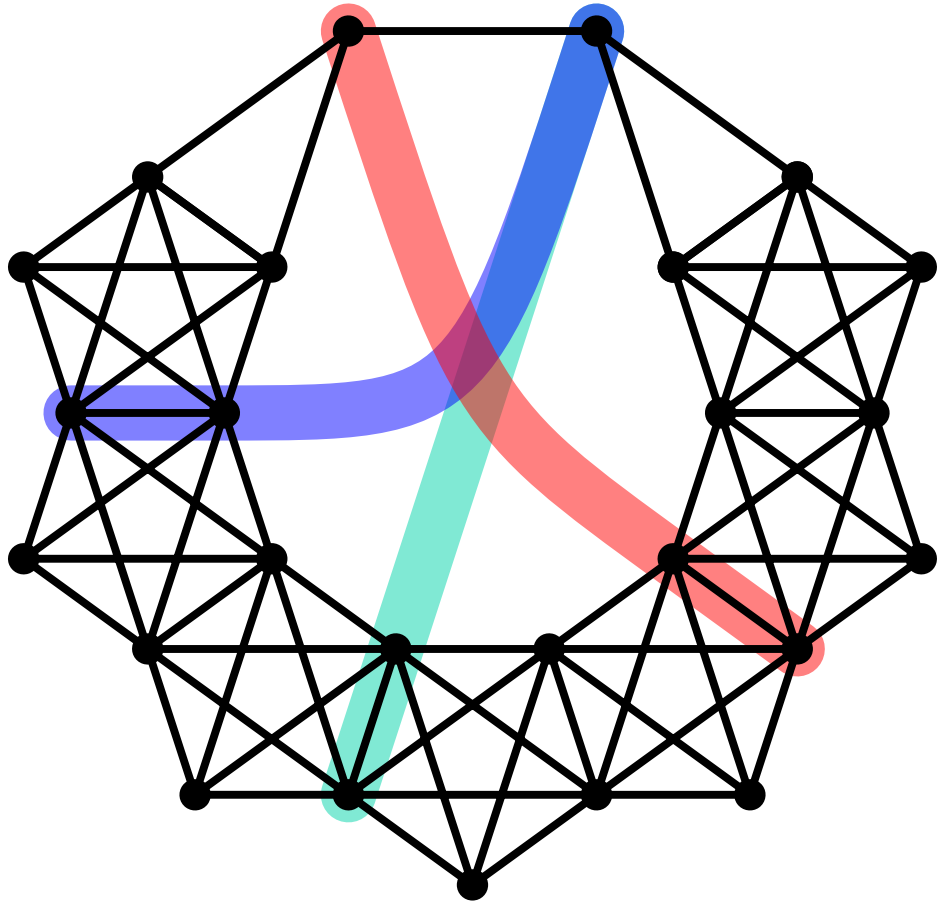

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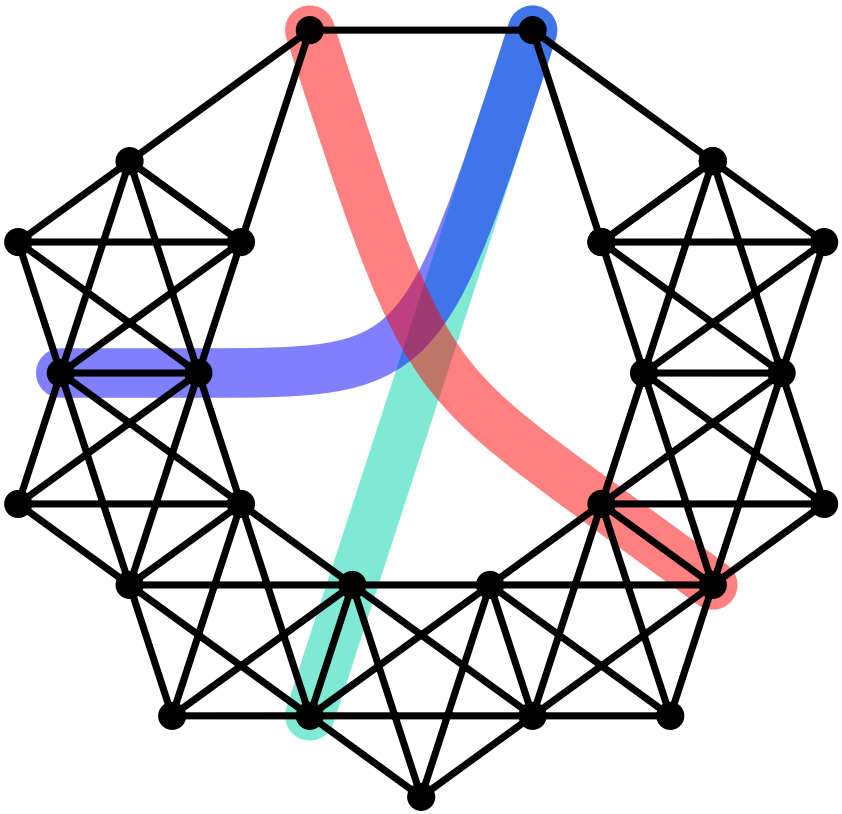

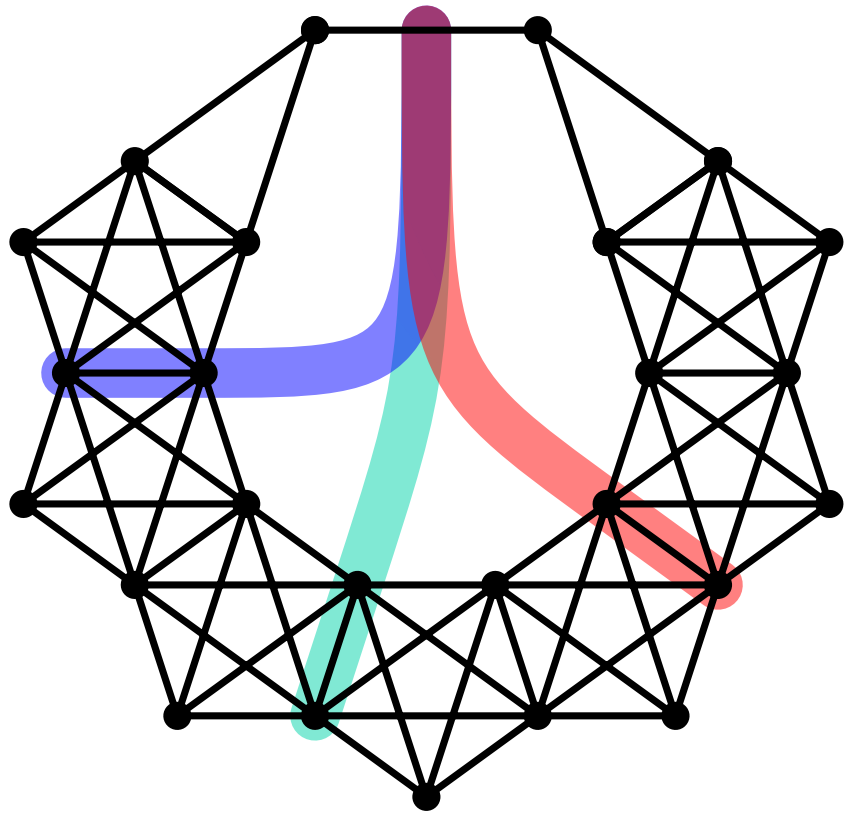

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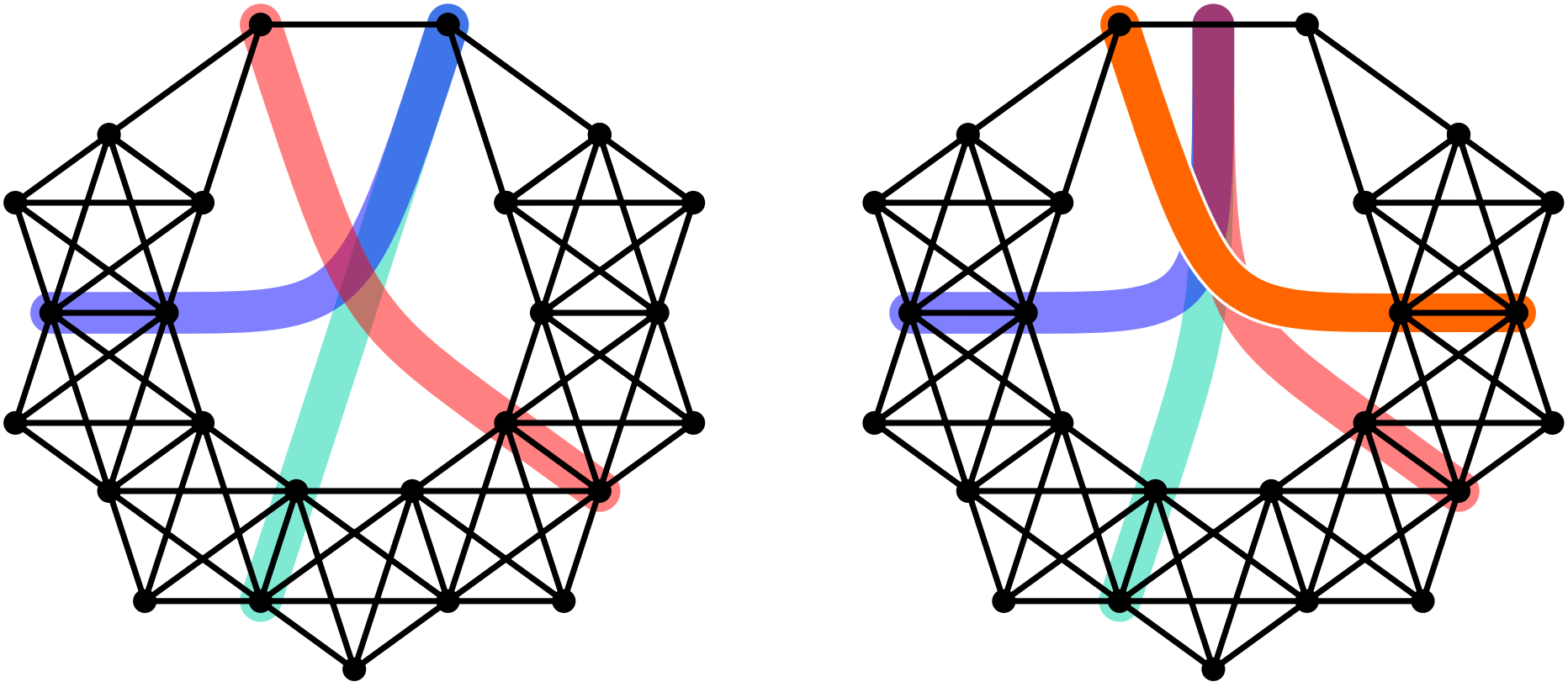

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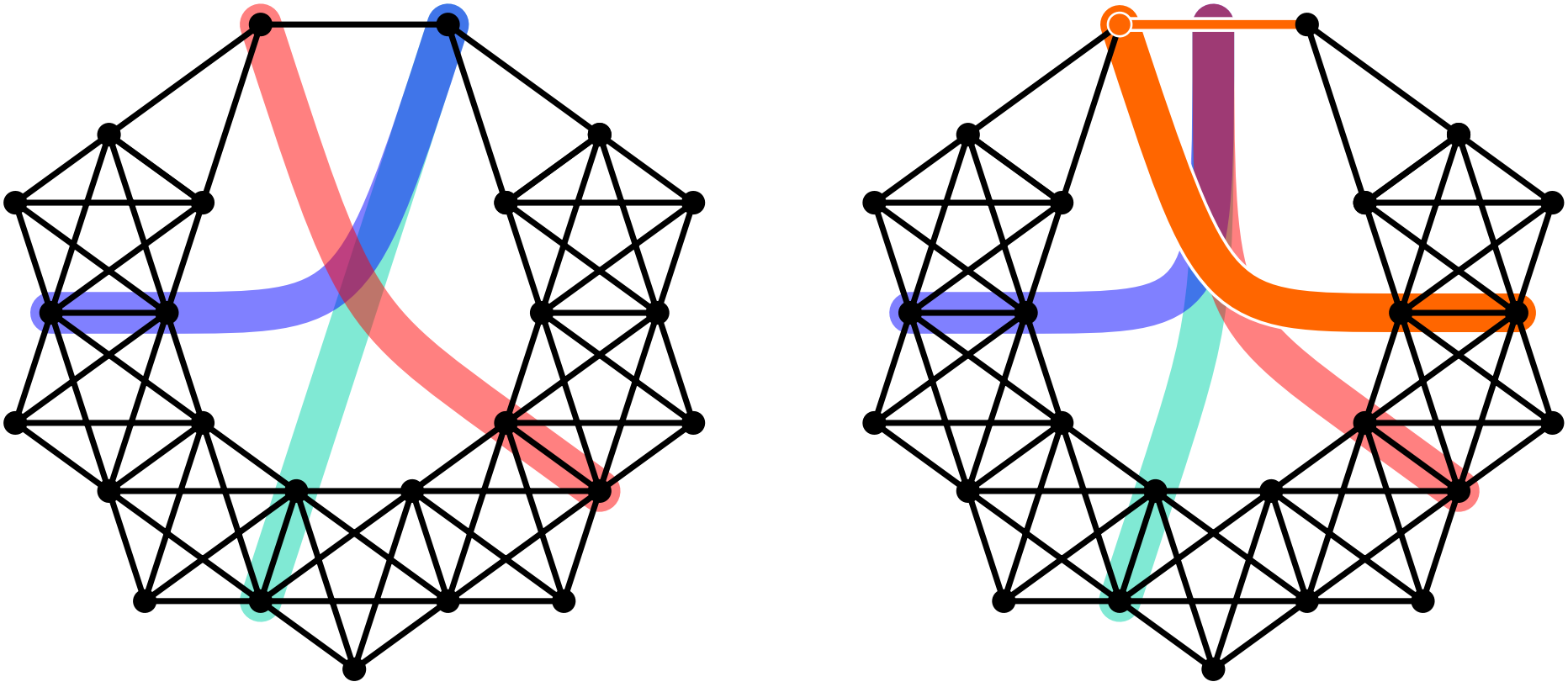

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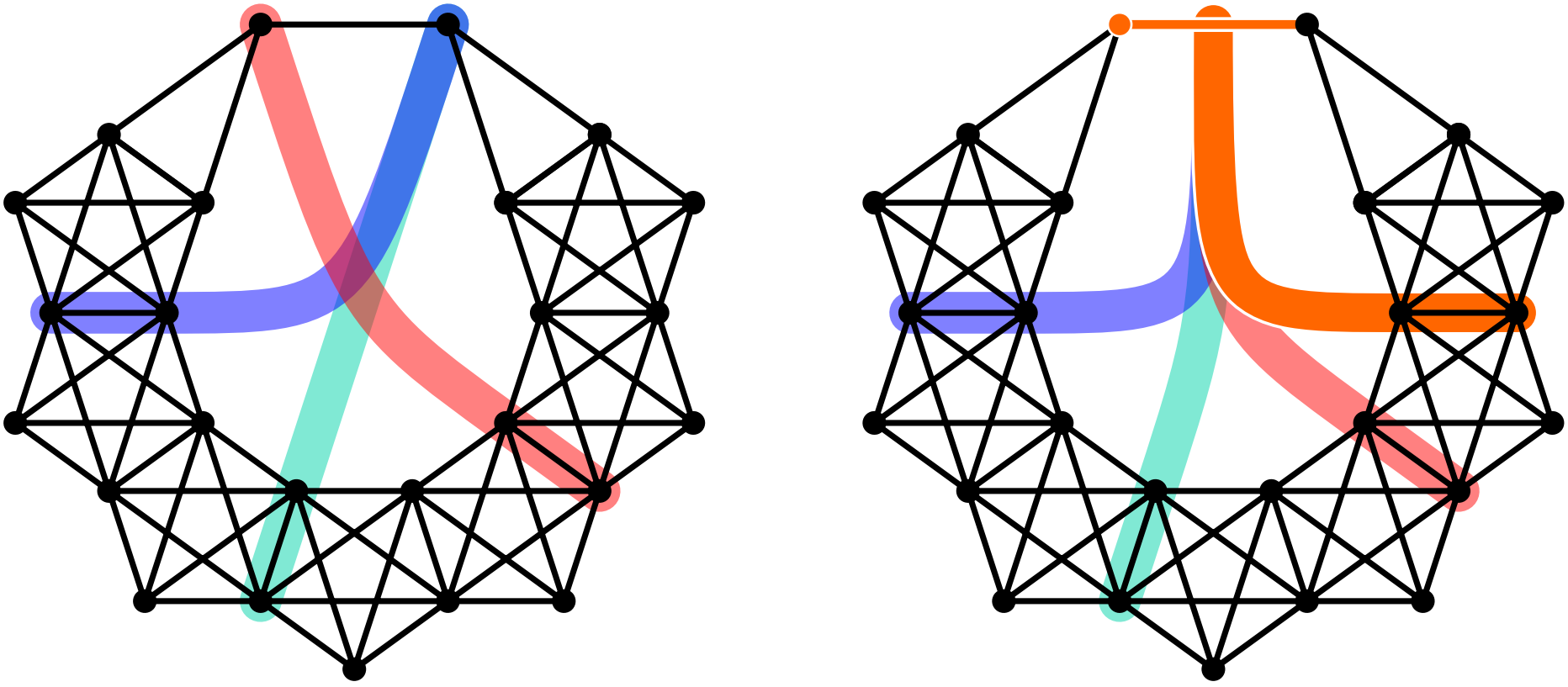

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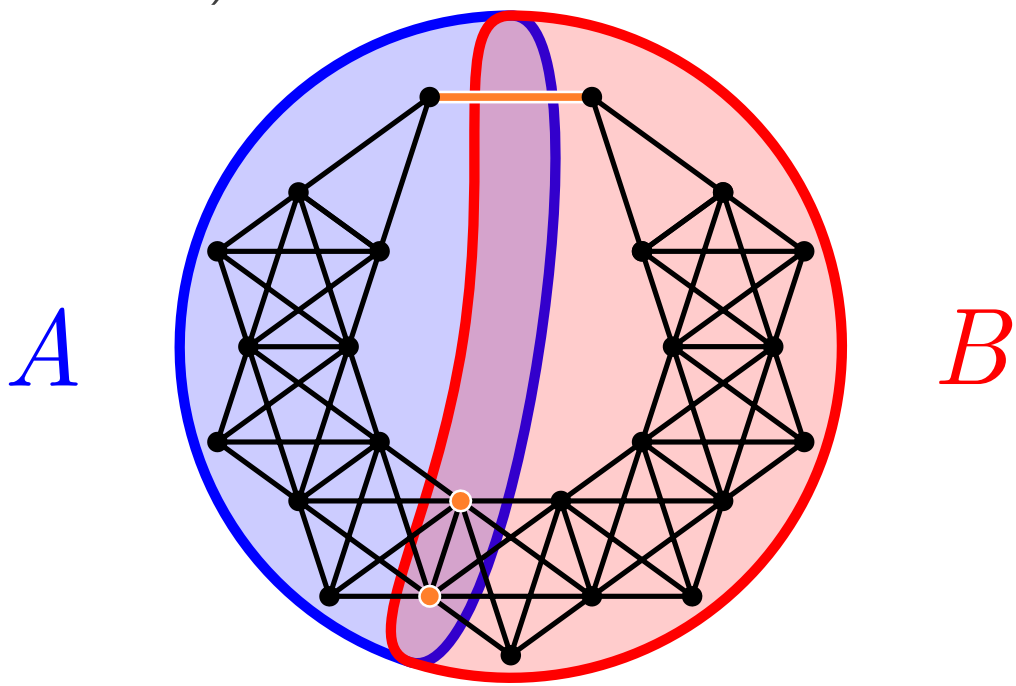

mixed-separation of G: (A, B) with $A \cup B = V(G)$ and $A, B \neq V(G)$

separator of (A, B): $(A \cap B) \cup E(A \smallsetminus B, B \smallsetminus A)$

A *tri-separation* of G is a mixed-sep'n (A, B) with $|\text{sep'r}| = 3$ s.t. every vx in $A \cap B$ has $\geqslant 2$ neighb's in A and B.

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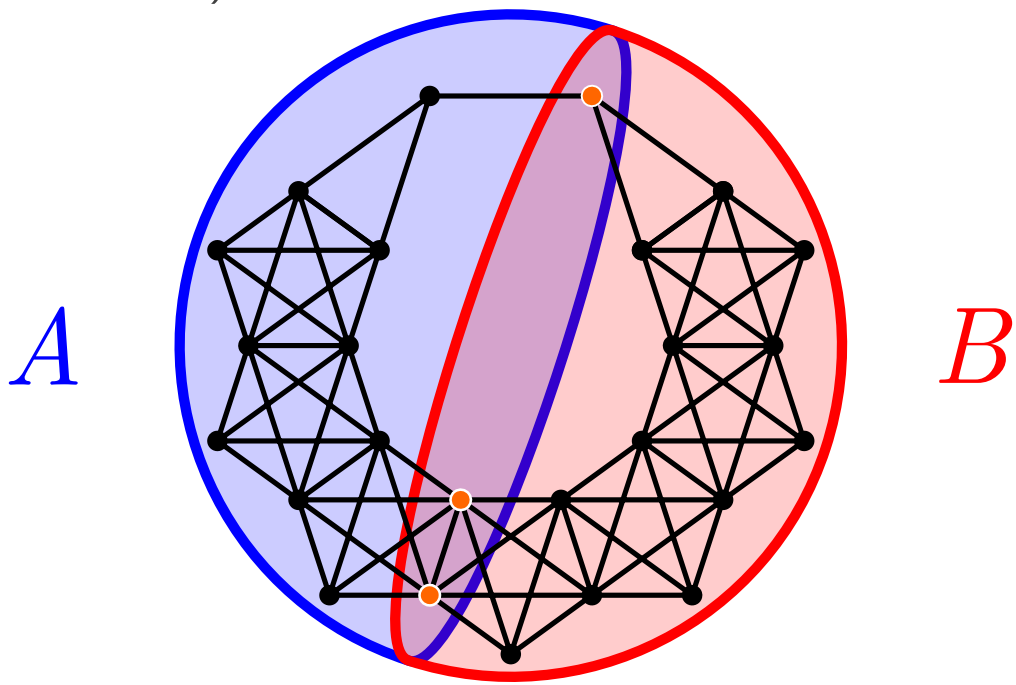

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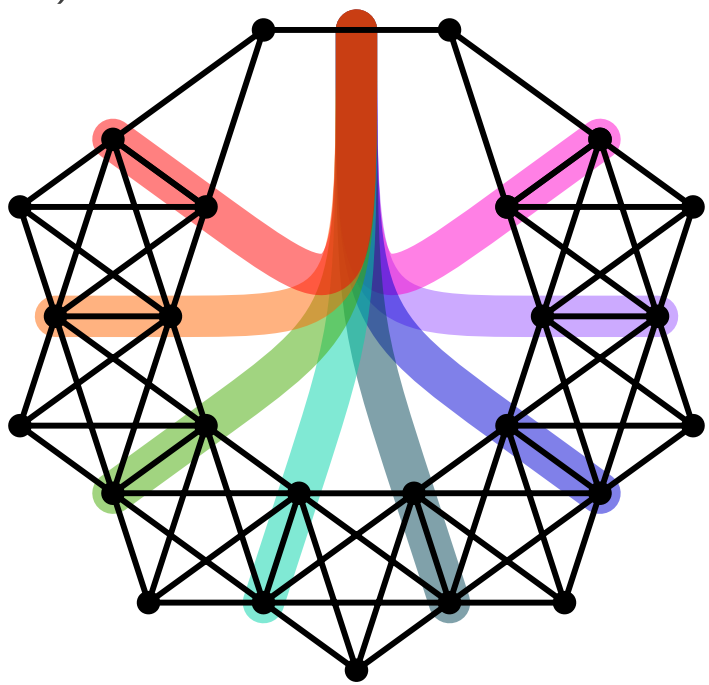

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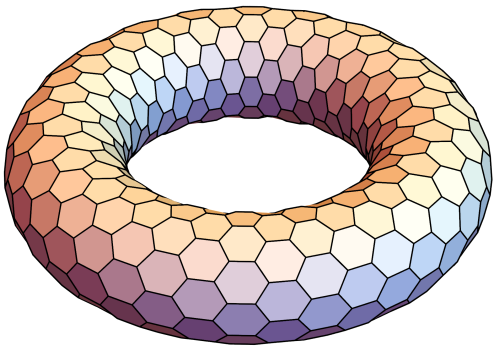
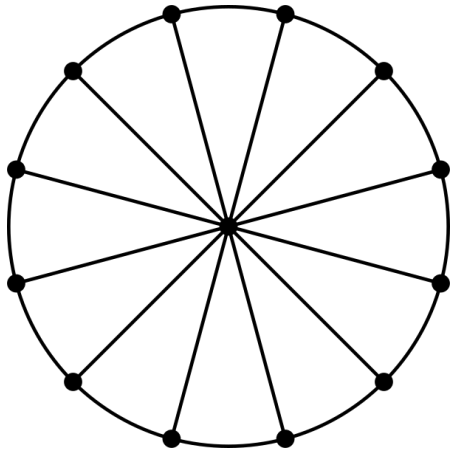
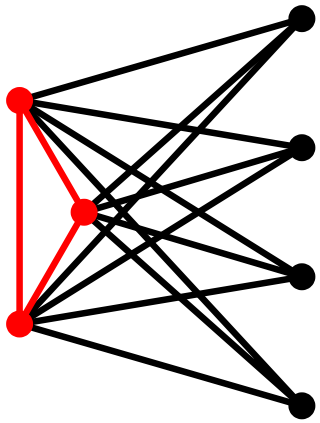
Theorem (Carmesin & K. 23)

Every 3-con'd G decomposes along its totally-nested nontrivial tri-separations into parts that are

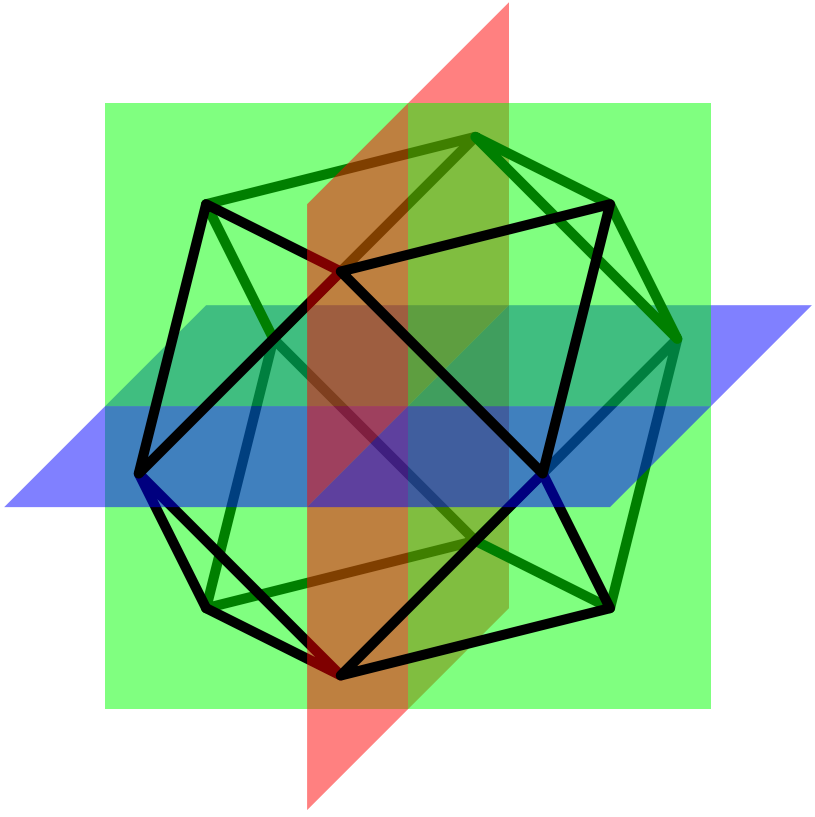
- quasi-4-con'd
- wheels
- thickened $K_{3,m}$

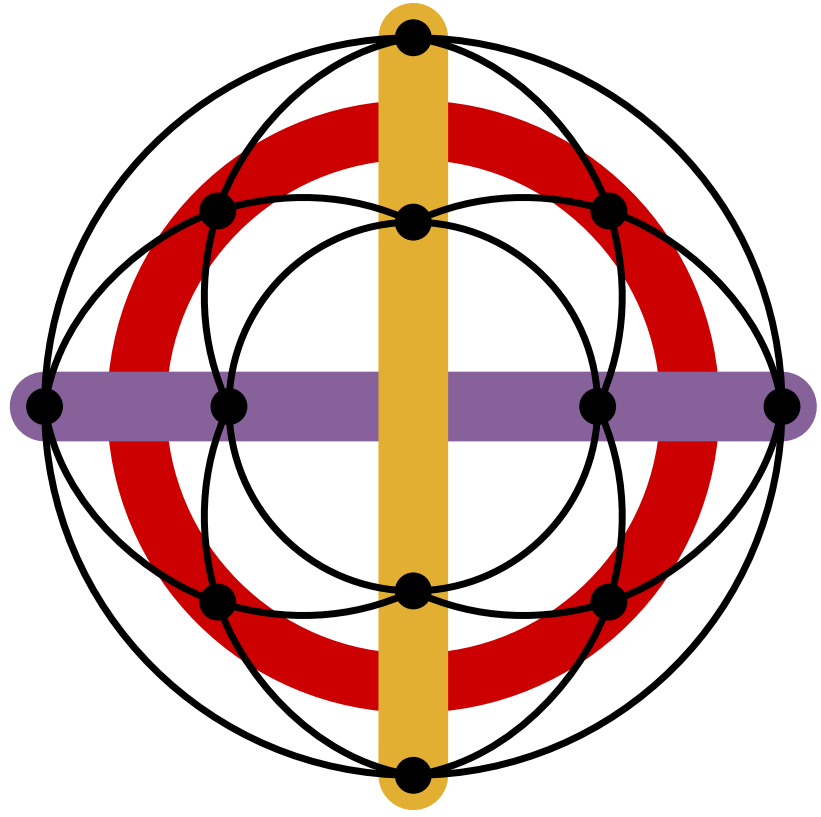






$k = 4?$

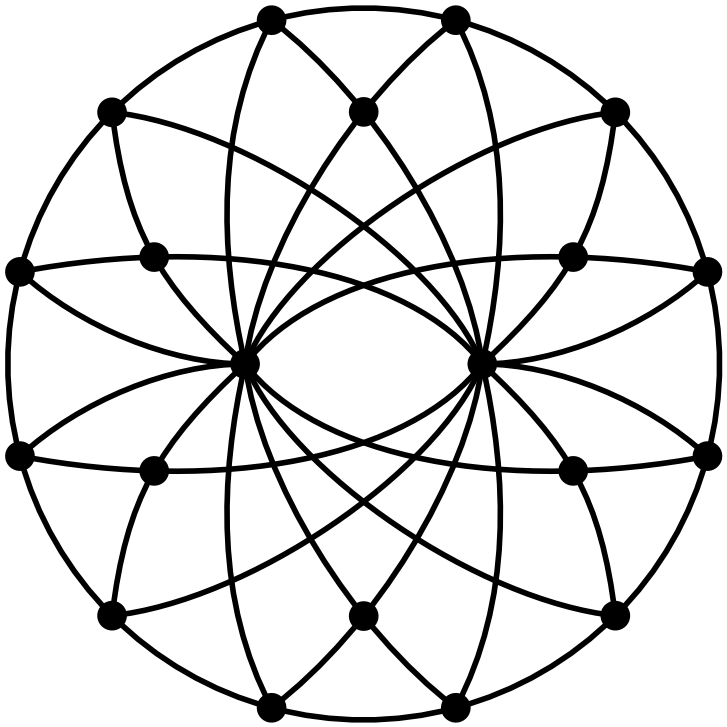


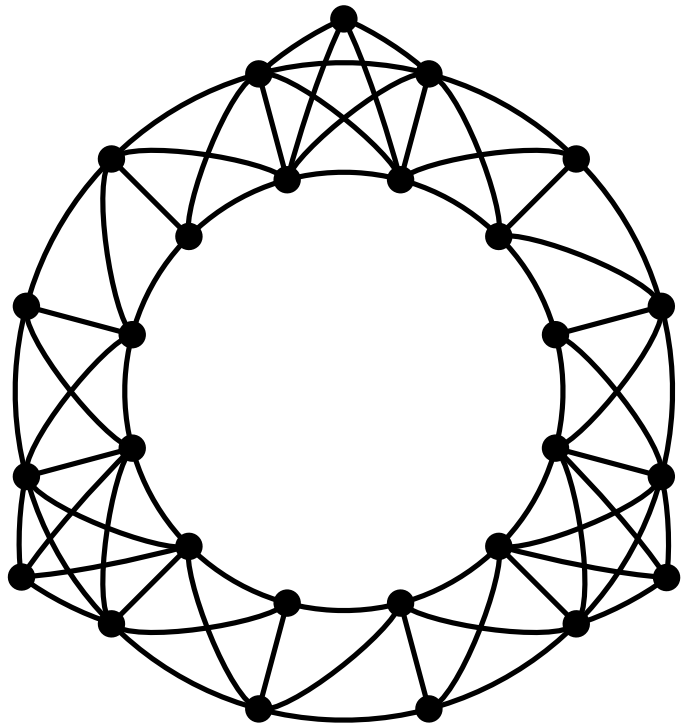


Theorem (K. & Planken 25)

Every 4-con'd G decomposes along its totally-nested tetra-separations into parts that are

- quasi-5-con'd
- generalised double-wheels
- thickened $K_{4,m}$
- cycles of triangles and 3-con'd graphs on $\leqslant 5$ vxs.

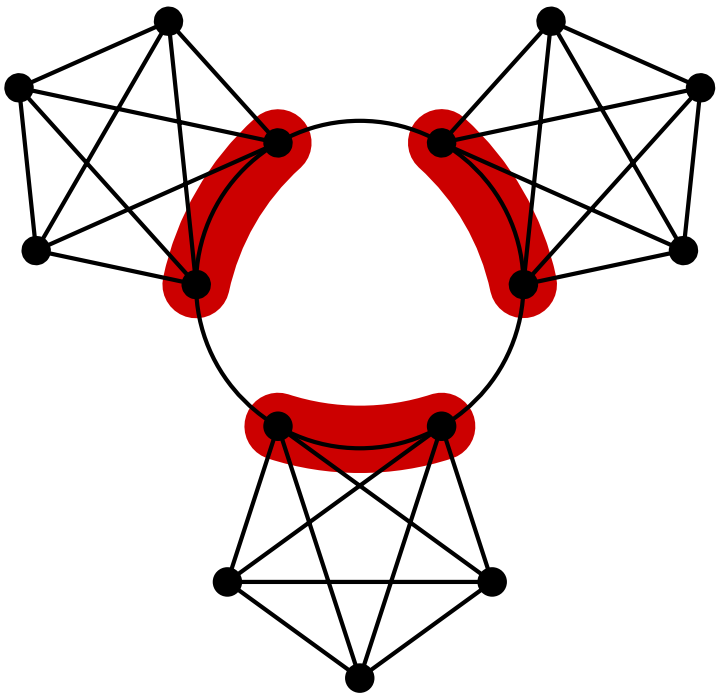


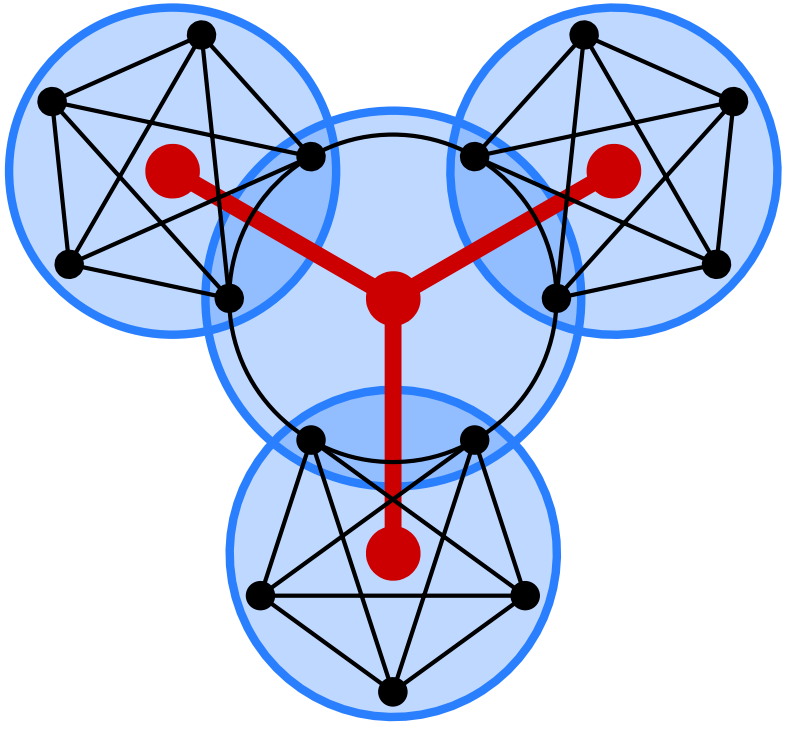


Takeaways from $k \leqslant 4$

Have to **relax** k-connectivity.

Can use **tree-decomposition**.



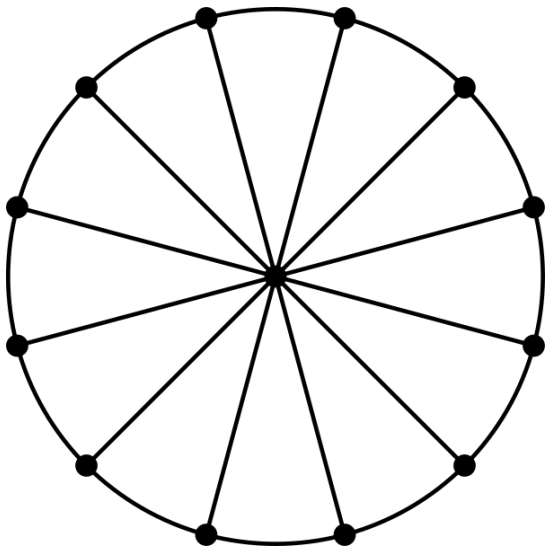


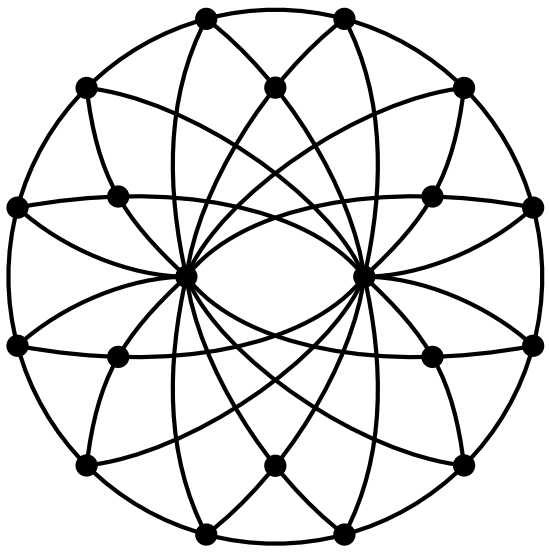
$(T, (V_t)_{t \in V(T)})$ where T is *decomposition-tree*, $V_t \subseteq V(G)$ are *bags*,

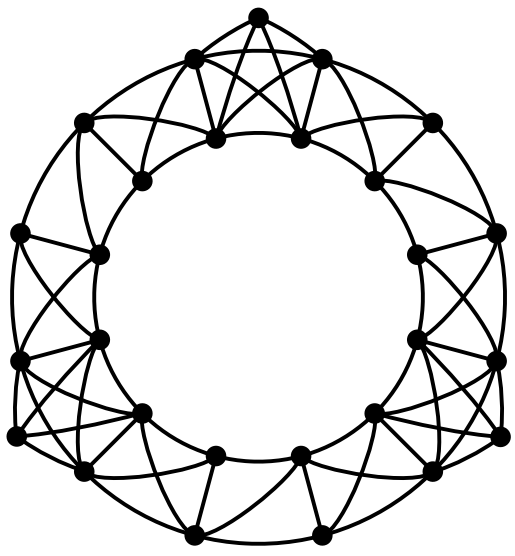
- $G = \bigcup \{ G[V_t] : t \in T \}$
- $\forall v \in V(G) : \quad \{ t \in T : v \in V_t \}$ is con'd in T.

Takeaways from $k \leqslant 4$

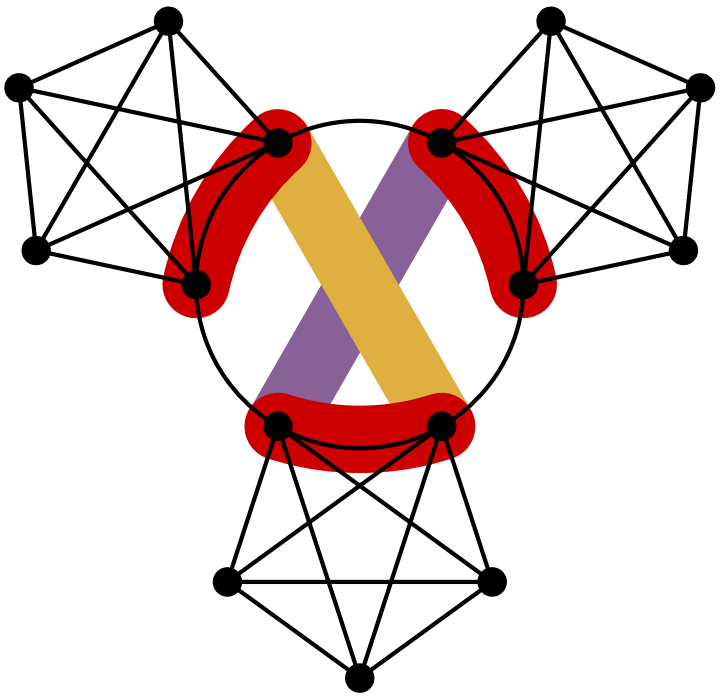
Basic pieces have **cyclic** structure.

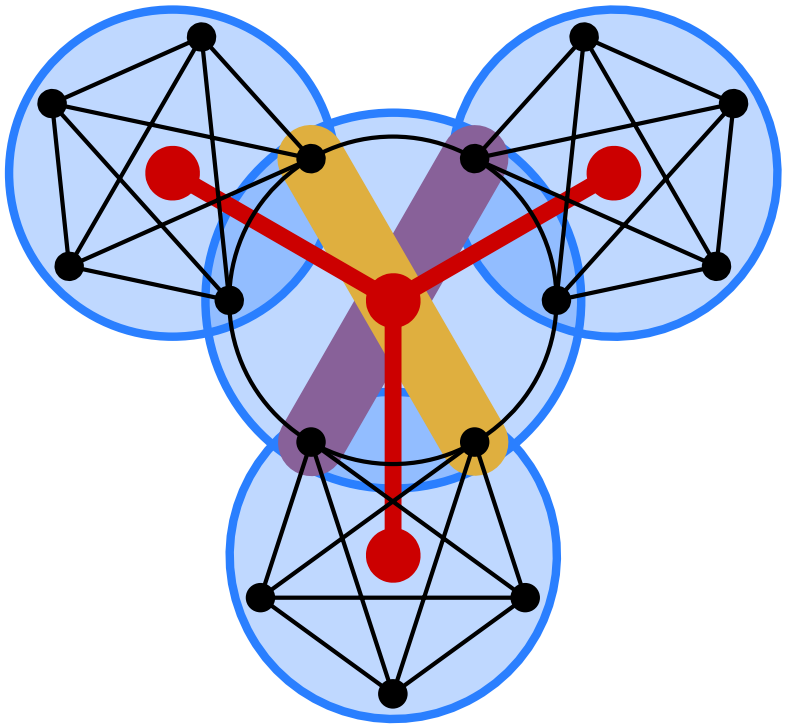






Can **display** all k-separations.





Wishful Thinking

Every approximately k-connected G has a tree-decomposition $(T, \mathcal{V})$ along approximately totally-nested k-separations such that:

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Every approximately k-connected G has a tree-decomposition $(T, \mathcal{V})$ along approximately totally-nested k-separations such that:

- every torso at a node $t \in T$ is either approximately $(k+1)$-connected or approximately has a cyclic structure F_t;

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- every k-separation of G is approximately displayed either by an edge of $(T, \mathcal{V})$ or by some F_t;

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- $(T, \mathcal{V})$ is approximately canonical.

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Plan for today: make this into theorem

Wishful Thinking

Every approximately k-connected G has a tree-decomposition $(T, \mathcal{V})$ along approximately totally-nested k-separations such that:

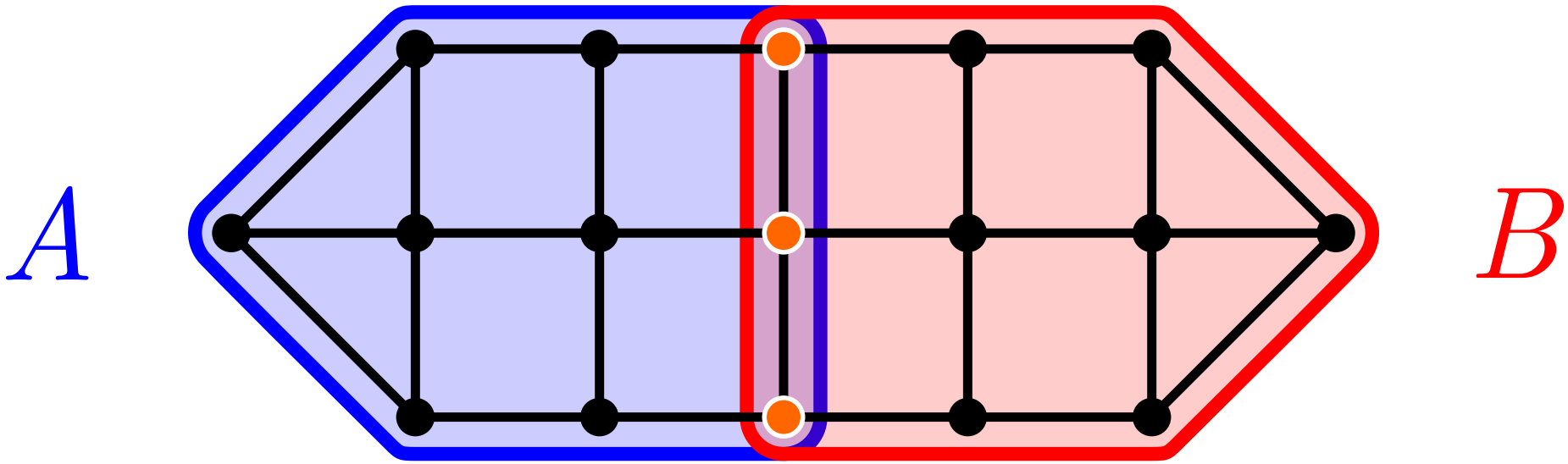
- every torso at a node $t \in T$ is either approximately $(k+1)$-connected or approximately has a cyclic structure F_t;
- every k-separation of G is approximately displayed either by an edge of $(T, \mathcal{V})$ or by some F_t;
- $(T, \mathcal{V})$ is approximately canonical.

Plan for today: make this into theorem

The *breadth* of (A, B) is $\min\{\,|A \smallsetminus B|, |B \smallsetminus A|\,\}$.

(A, B) is *β-broad* $:\Longleftrightarrow$ (A, B) has breadth $\geqslant \beta$.

G is *q-almost-k-connected* $:\Longleftrightarrow$ every $(< k)$-sep'n has breadth $\leqslant q$.



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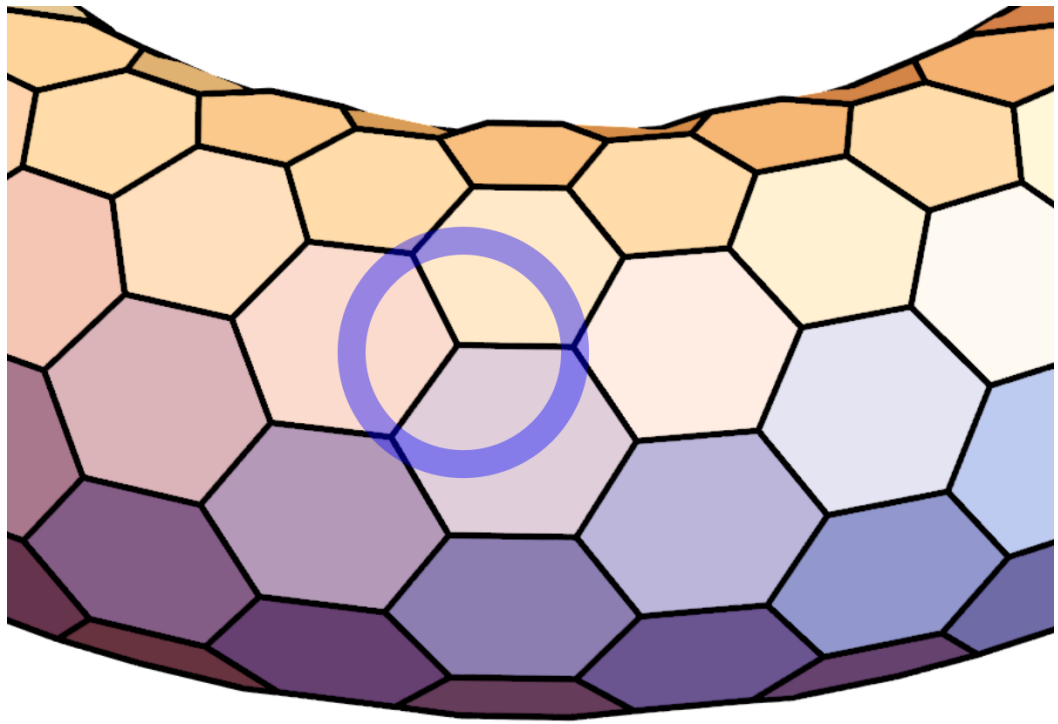

1-almost-4-con'd

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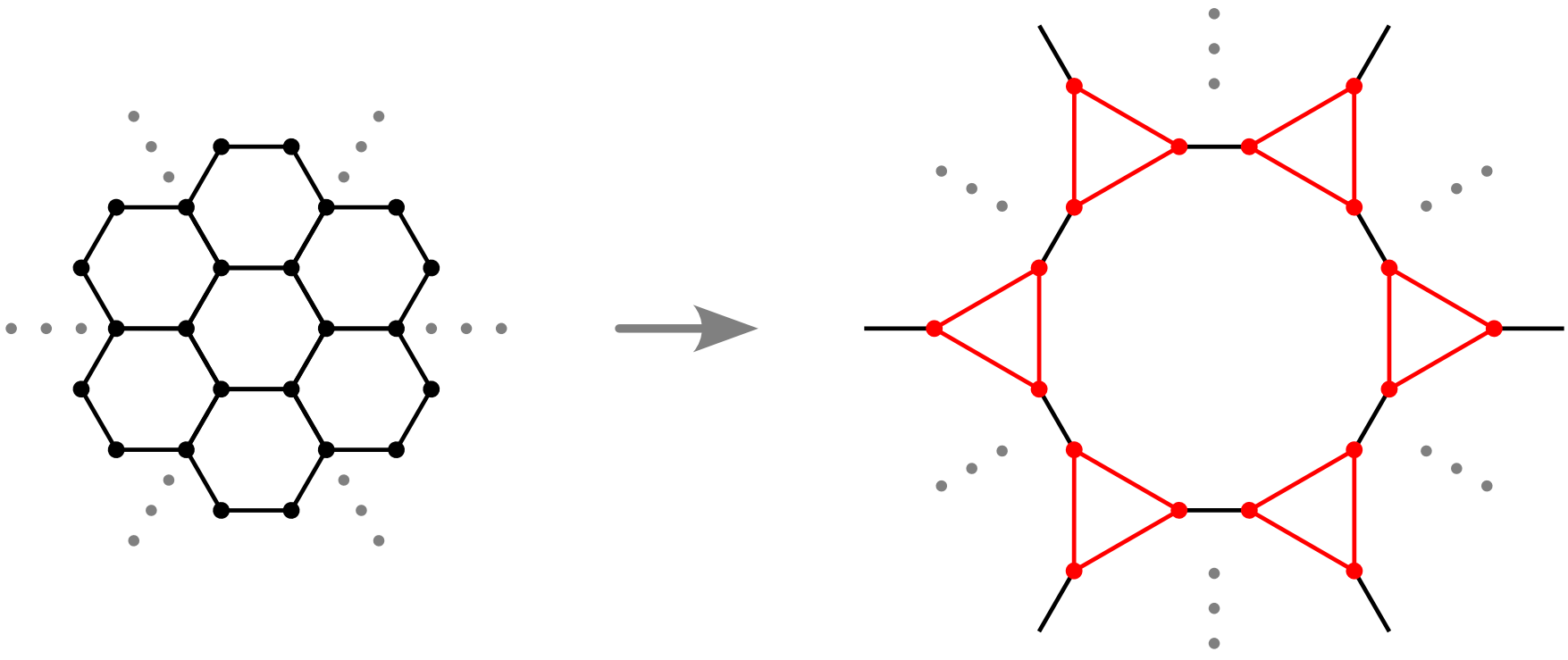

3-almost-4-con'd

Wishful Thinking

Every q-almost-k-connected G has a tree-decomposition $(T, \mathcal{V})$ along approximately totally-nested k-separations such that:

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Wishful Thinking

For every $q, k \in \mathbb{N}$ there is $q' \in \mathbb{N}$ such that the following holds.

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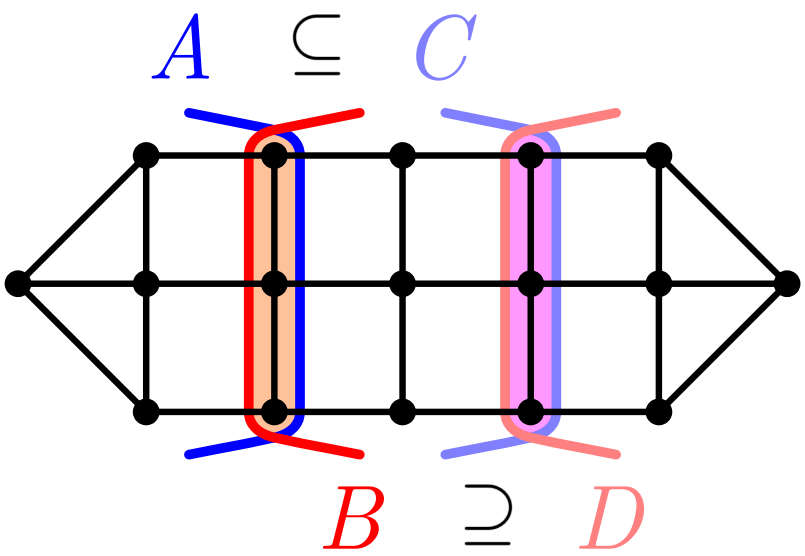
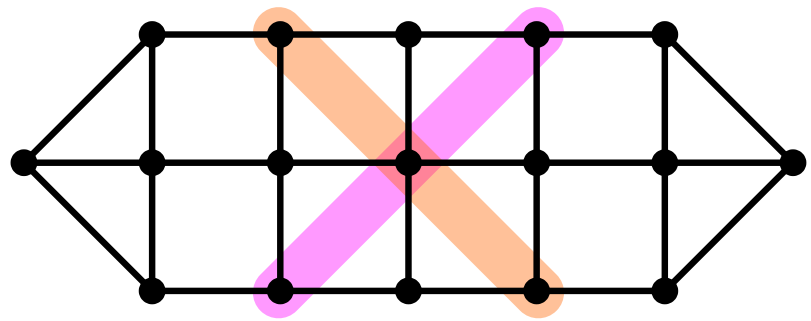
- $(T, \mathcal{V})$ is approximately canonical.

$(A, B) \leqslant (C, D) :\Longleftrightarrow A \subseteq C$ and $B \supseteq D$

(A, B) and (C, D) are *nested* $:\Longleftrightarrow (A, B) \leqslant (C, D)$ after possibly swapping names A with B or C with D

cross $:\Longleftrightarrow$ not nested

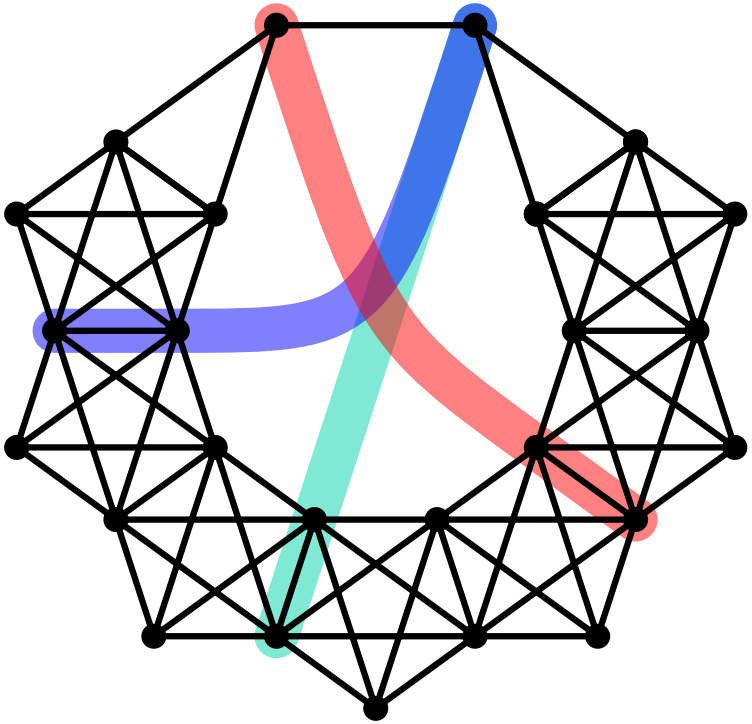




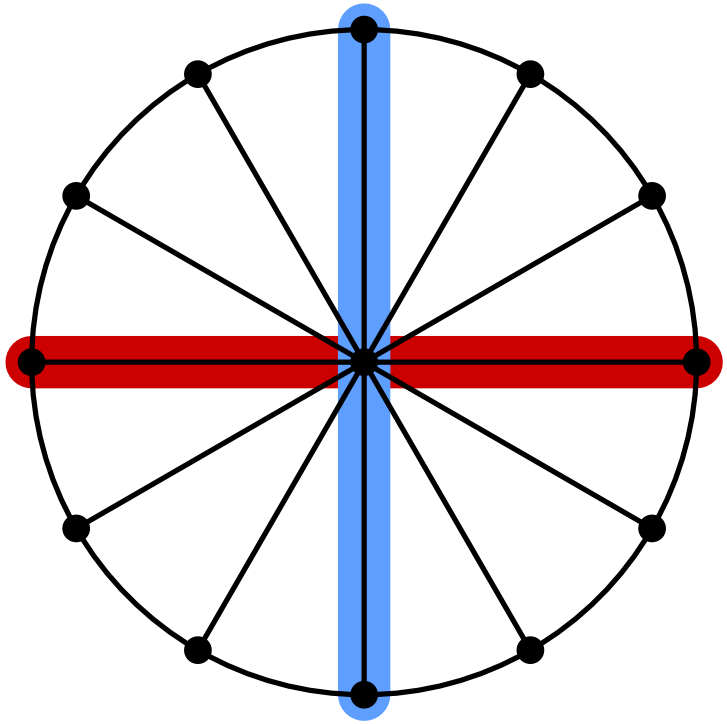
$(A, B) \leqslant_\varepsilon (C, D) :\Longleftrightarrow A \subseteq_\varepsilon C$ and $B \supseteq_\varepsilon D$, where
$X \subseteq_\varepsilon Y :\Longleftrightarrow |X \smallsetminus Y| \leqslant \varepsilon$

(A, B) and (C, D) are *ε-nested* $:\Longleftrightarrow (A, B) \leqslant_\varepsilon (C, D)$ after possibly swapping names A with B or C with D

ε-cross $:\Longleftrightarrow$ not ε-nested

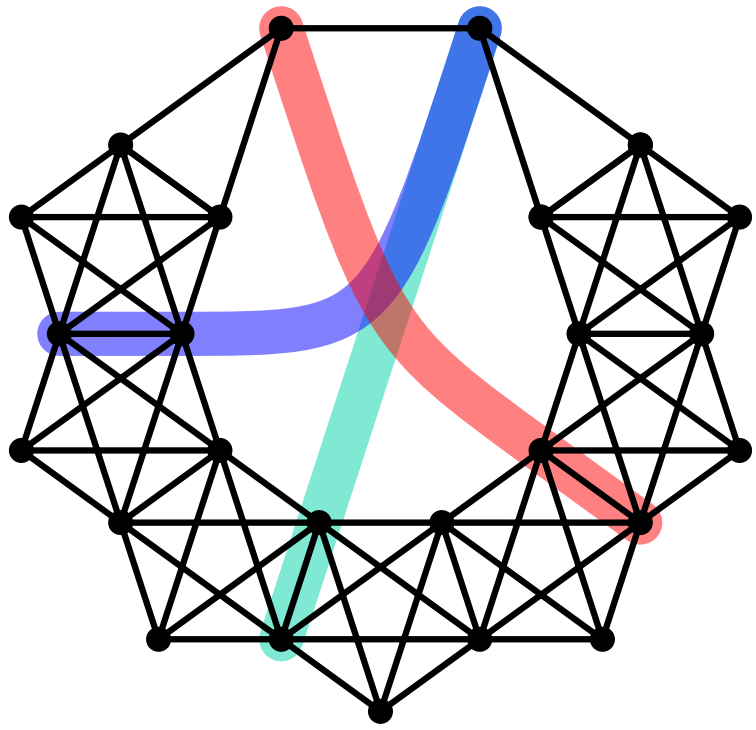


1-nested



2-crossed

A proper $(\leqslant k)$-separation is *totally-ε-nested* $:\Longleftrightarrow$
it is ε-nested with every proper $(\leqslant k)$-separation (of G)



totally-1-nested

Wishful Thinking

For every $q, k \in \mathbb{N}$ there are $q', \gamma \in \mathbb{N}$ such that the following holds.

Every q-almost-k-connected G has a tree-decomposition $(T, \mathcal{V})$ along totally-γ-nested $(\leqslant k)$-separations such that:

- every torso at a node $t \in T$ is either q'-almost-$(k+1)$-connected or approximately has a cyclic structure F_t;
- every k-separation of G is approximately displayed either by an edge of $(T, \mathcal{V})$ or by some F_t;
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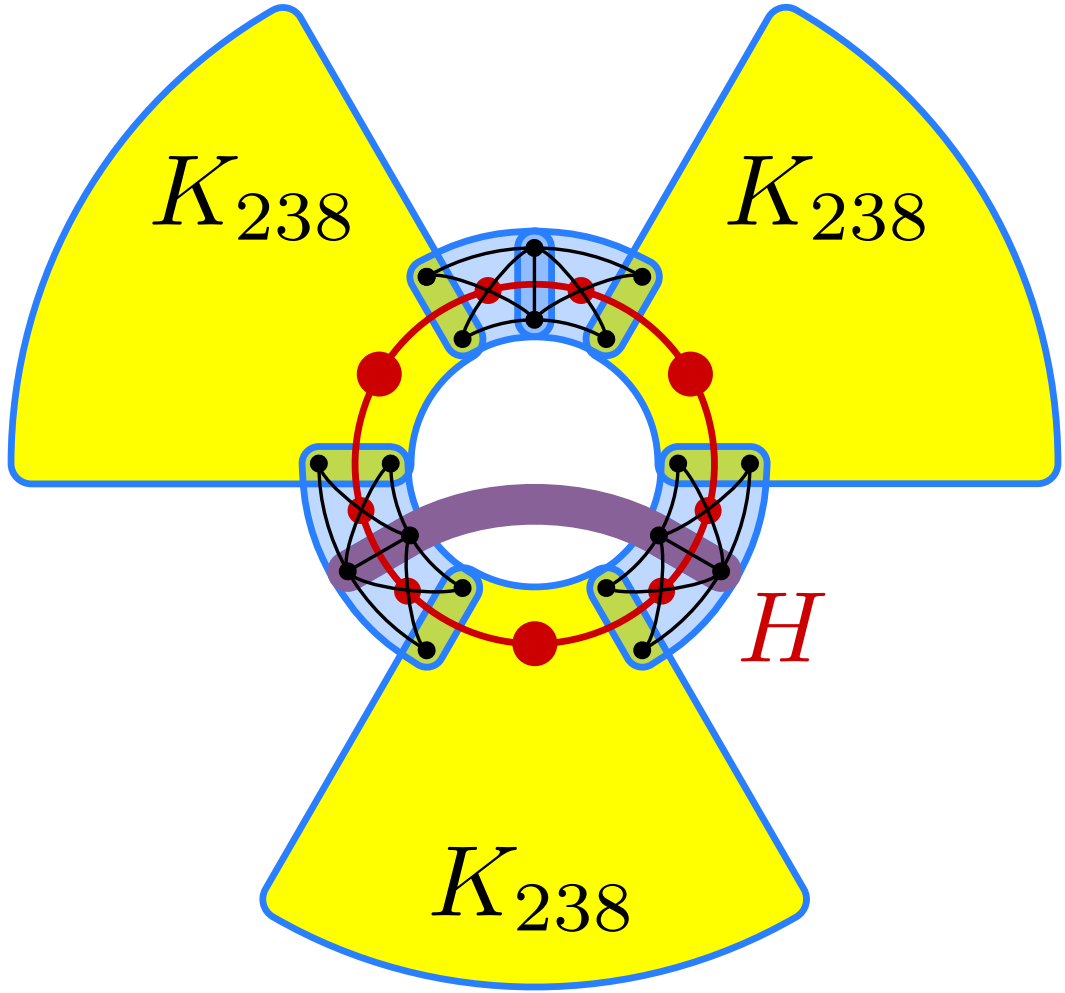
Let $(H, \mathcal{V})$ be a **cycle**-decomposition of G.

Every bipartition $b = (X, Y)$ of $V(H)$ *induces* the separation $s(b) := (\bigcup_{x \in X} V_x, \bigcup_{y \in Y} V_y)$ of G.

If $H[X]$ and $H[Y]$ are paths, then $s(b)$ is *displayed* by $(H, \mathcal{V})$.



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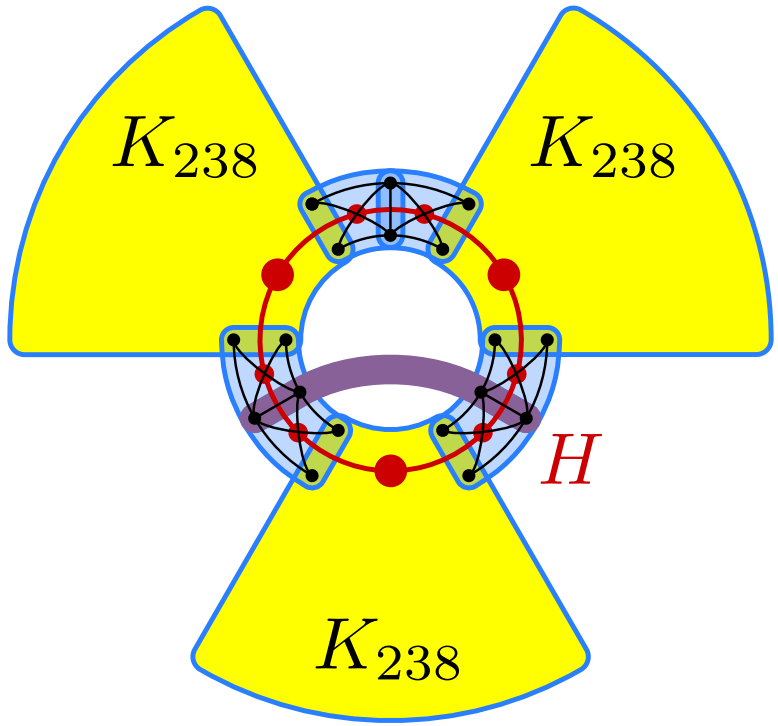
If $H[X]$ and $H[Y]$ are paths, then $s(b)$ is *displayed* by $(H, \mathcal{V})$.

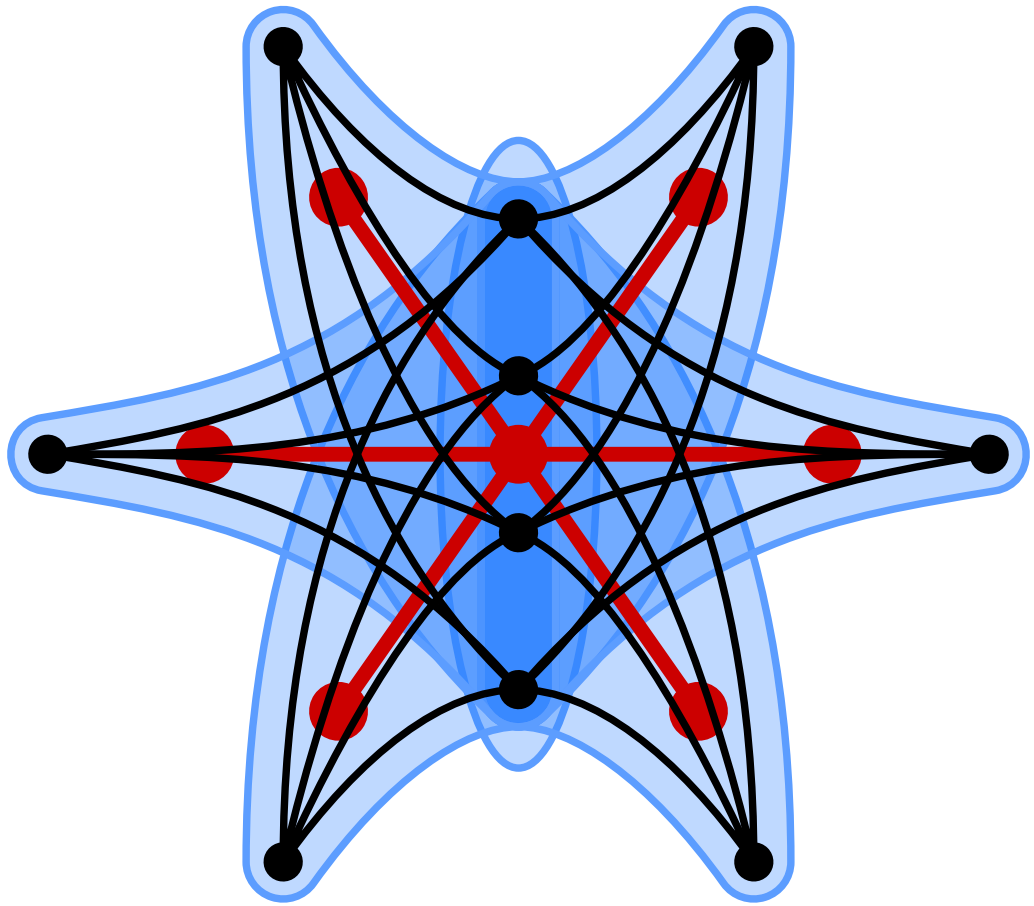
$(H, \mathcal{V})$ is *k-daisy* if

- every displayed separation has order $\leqslant k$, and
- every induced separation of order $\leqslant k$ is also displayed.

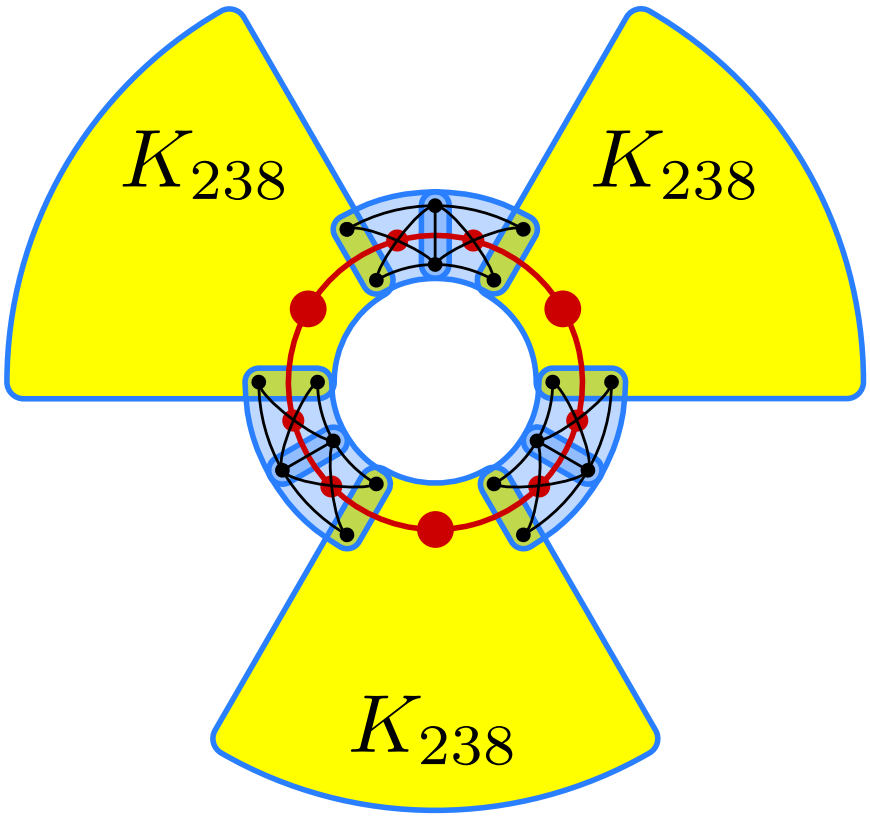


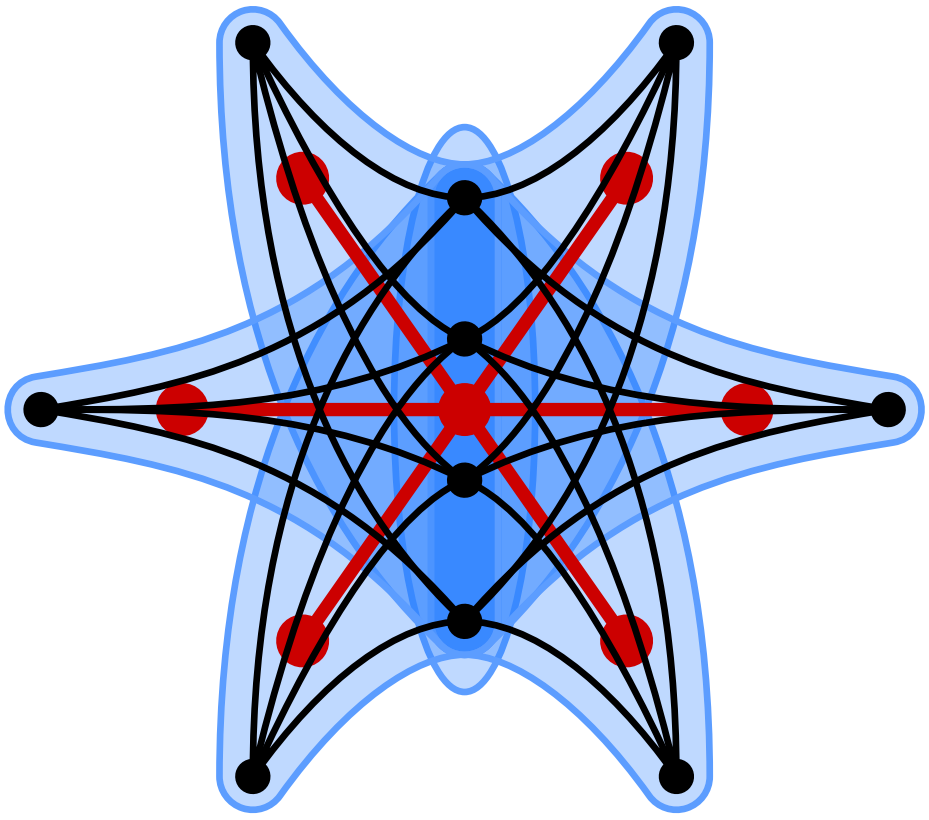
A *k-anemone* is a star-decomposition of G such that

- central bag has size $\leqslant k$, and
- every leaf-bag includes the central bag.



A *k-flower* is a k-daisy or a k-anemone.





Wishful Thinking

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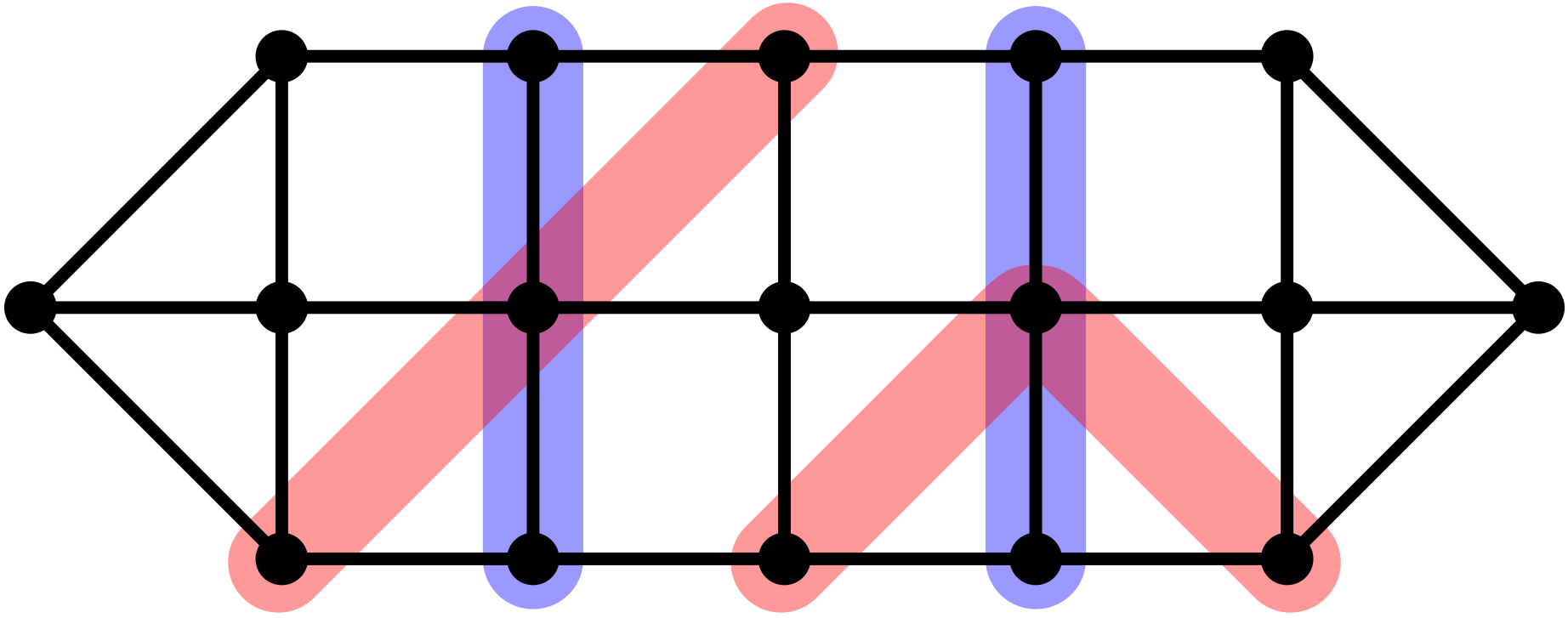
(A, B) and (C, D) are ε-close $:\Longleftrightarrow$ $(A, B) \leqslant_\varepsilon (C, D)$ and
$(A, B) \geqslant_\varepsilon (C, D)$

(Sufficient condition: $|A \triangle C| \leqslant \varepsilon$ and $|B \triangle D| \leqslant \varepsilon$.)



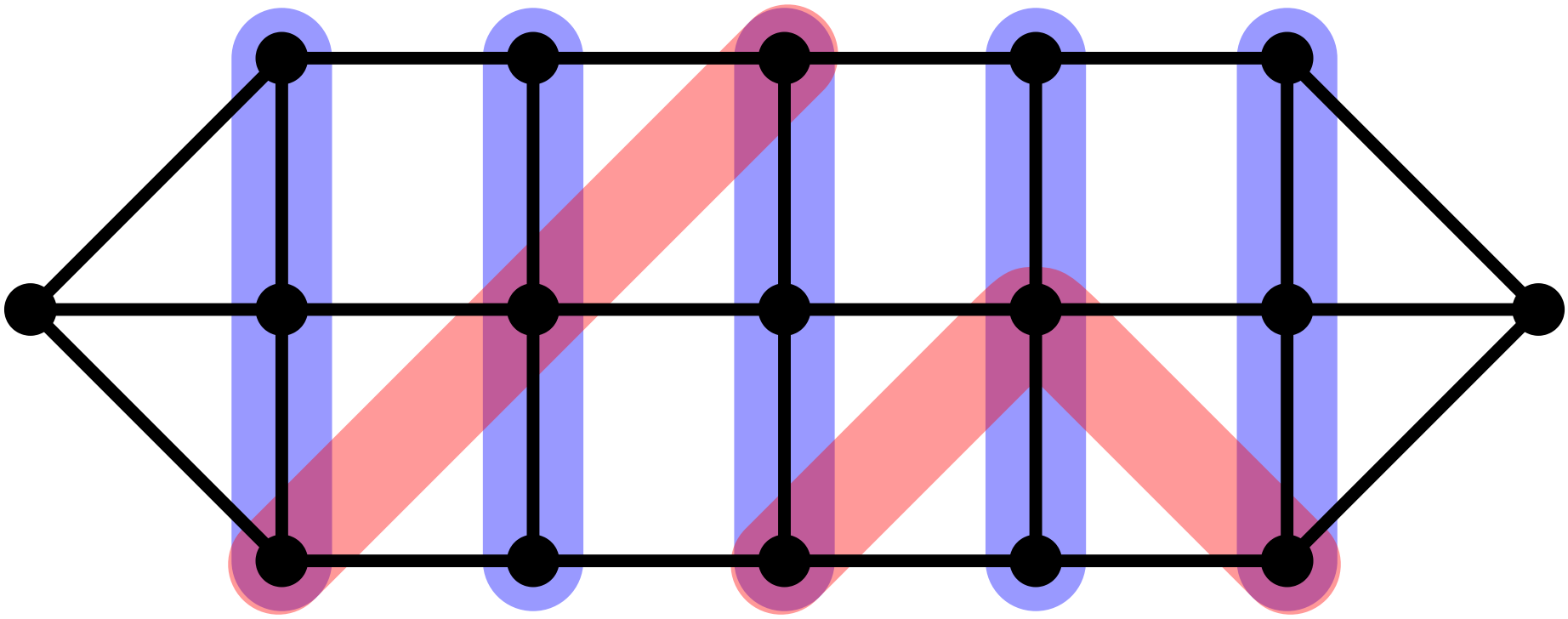
1-close

(2-close)

Let S, S' be sets of separations of G.

S *ε-approximates* $S' :\Longleftrightarrow$ every ε-broad $s' \in S'$ is ε-close to some $s \in S$.

Think: approximately $S \supseteq S'$

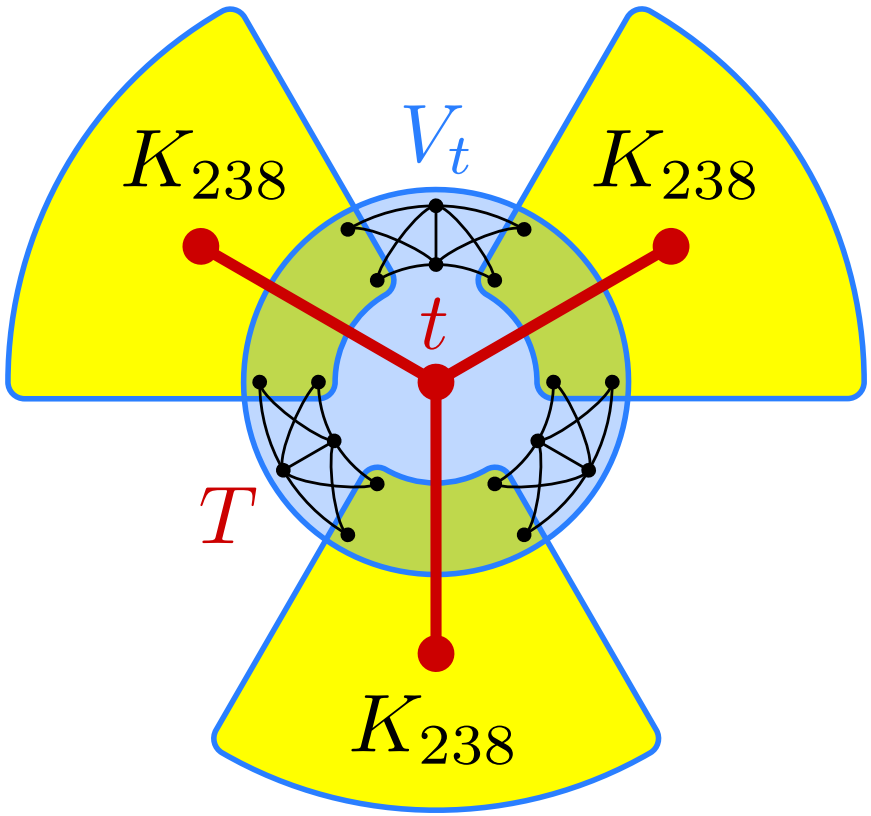


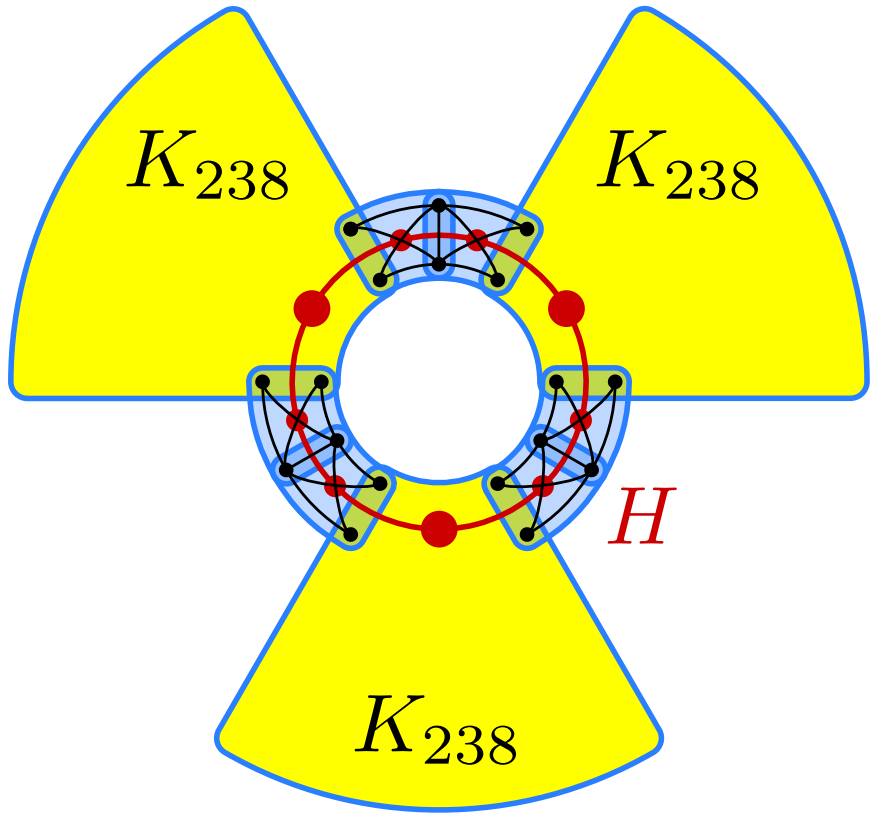
S 2-approximates the set of all 3-separations

Left: tree-decomposition $(T, \mathcal{V})$ of G

Right: k-daisy $F_t = (H, \mathcal{W})$ of G

F_t *ε-refines* $(T, \mathcal{V})$ at $t :\Longleftrightarrow \pi(F_t)$ ε-approximates σ_t





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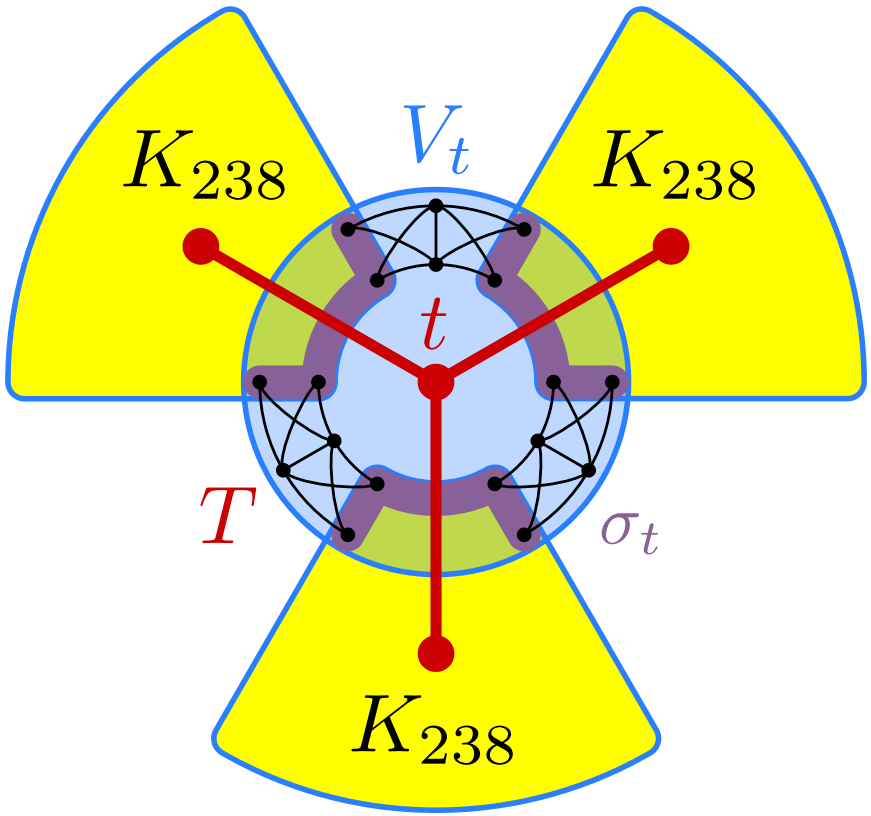

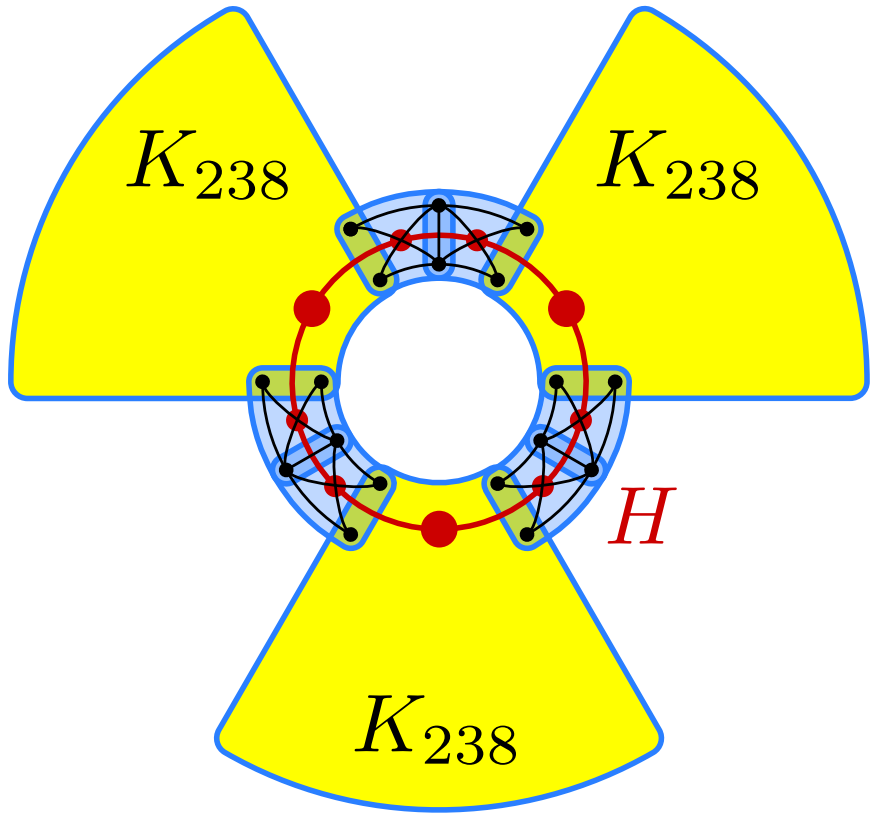

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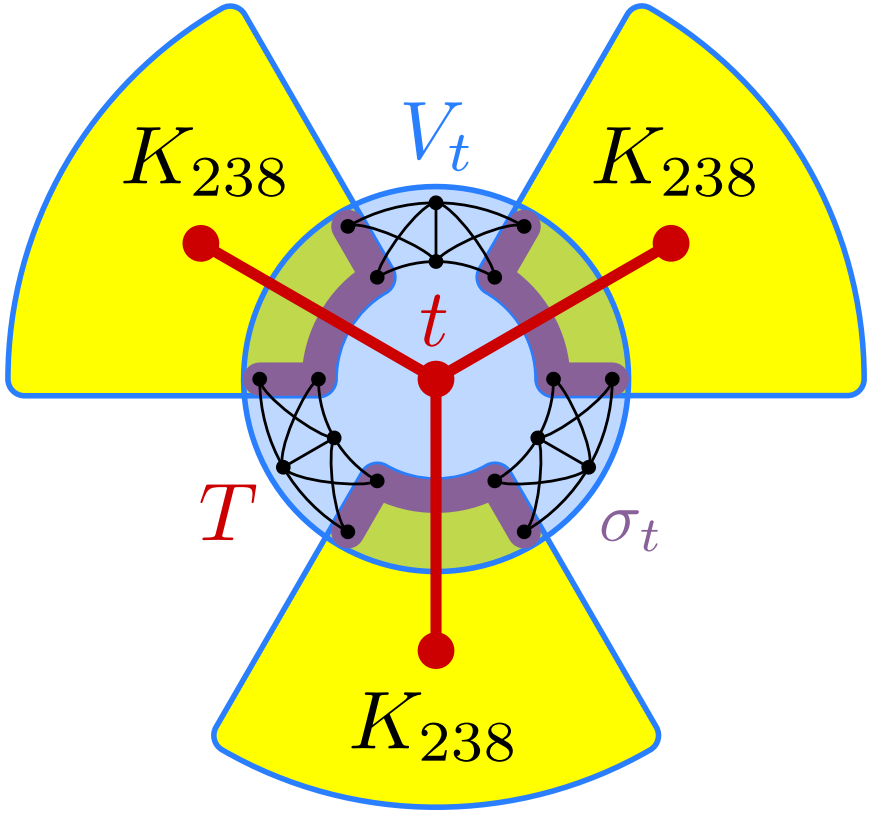

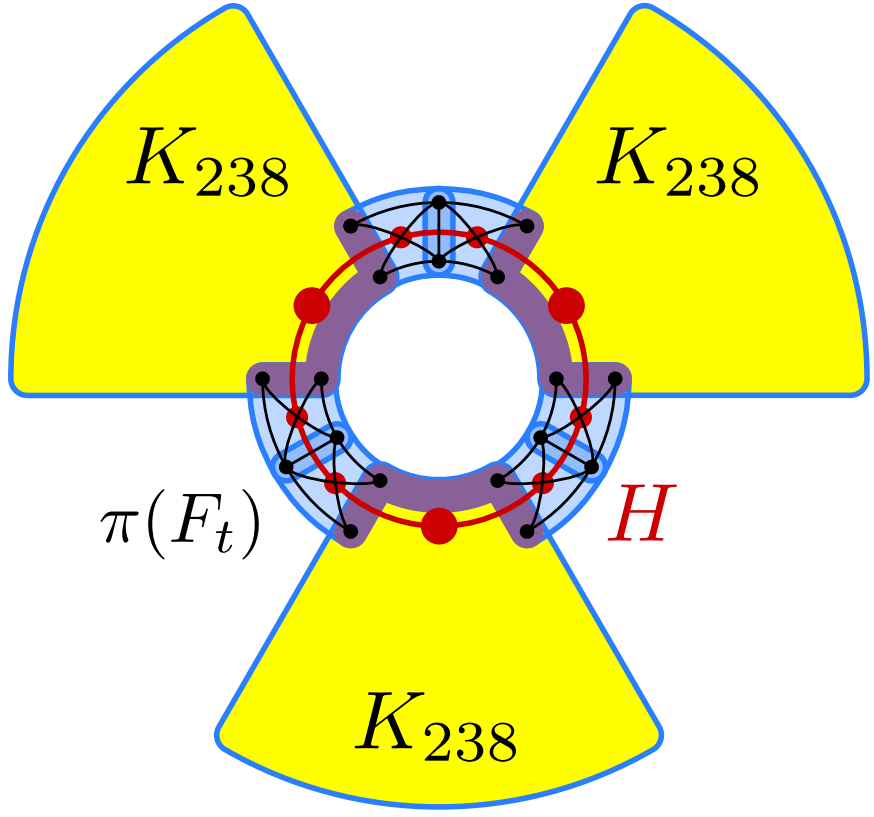

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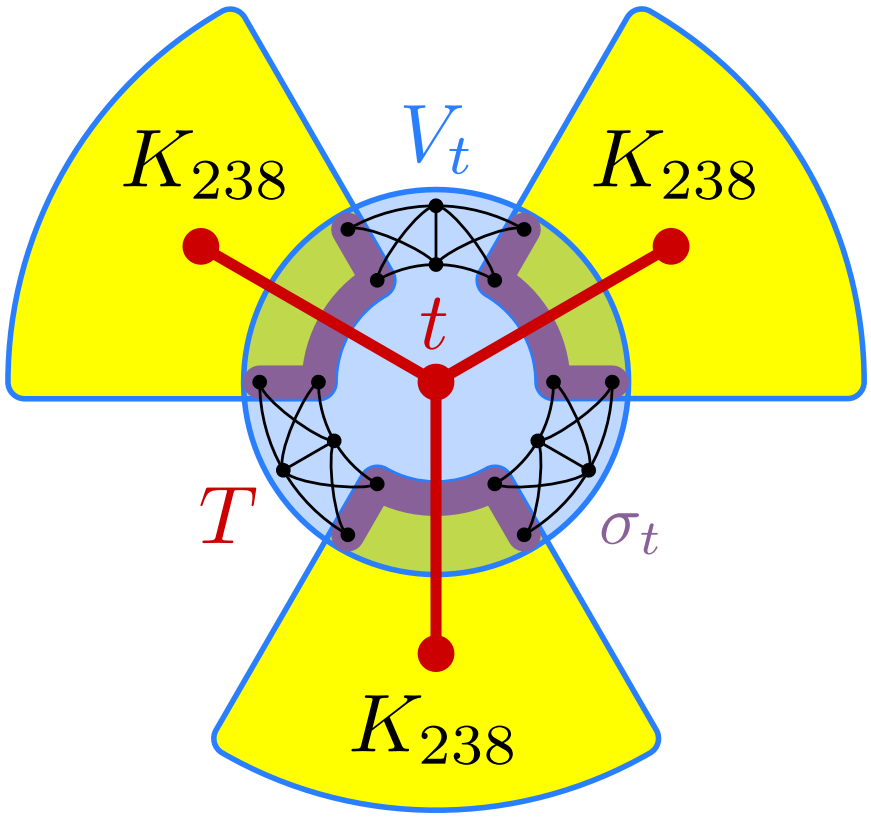

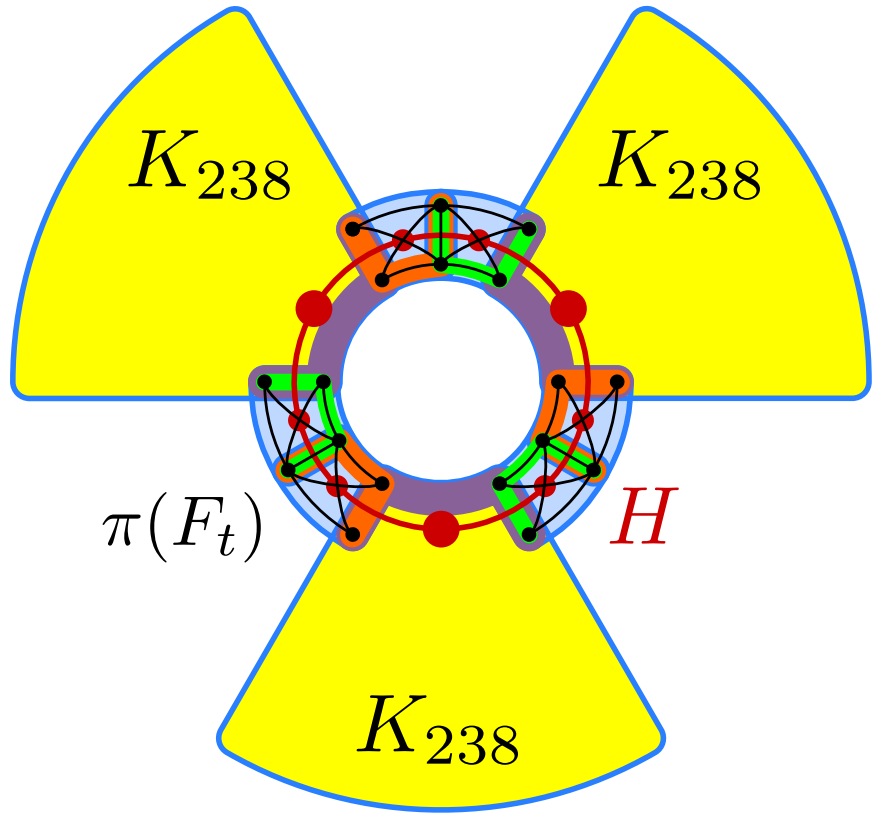

Wishful Thinking

For every $q, k \in \mathbb{N}$ there are $q', \gamma, \varepsilon \in \mathbb{N}$ such that the following holds.

Every q-almost-k-connected G has a tree-decomposition $(T, \mathcal{V})$ along totally-γ-nested $(\leqslant k)$-separations such that:

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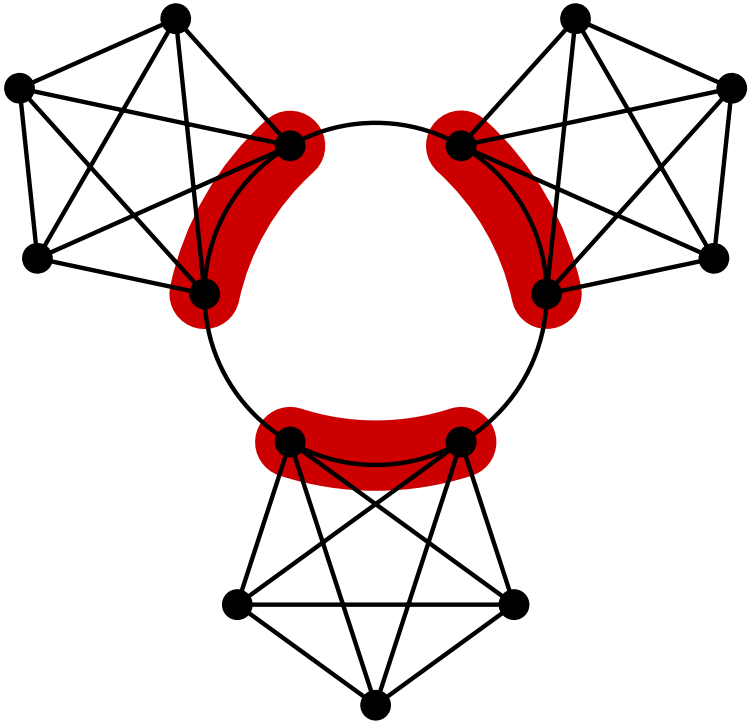
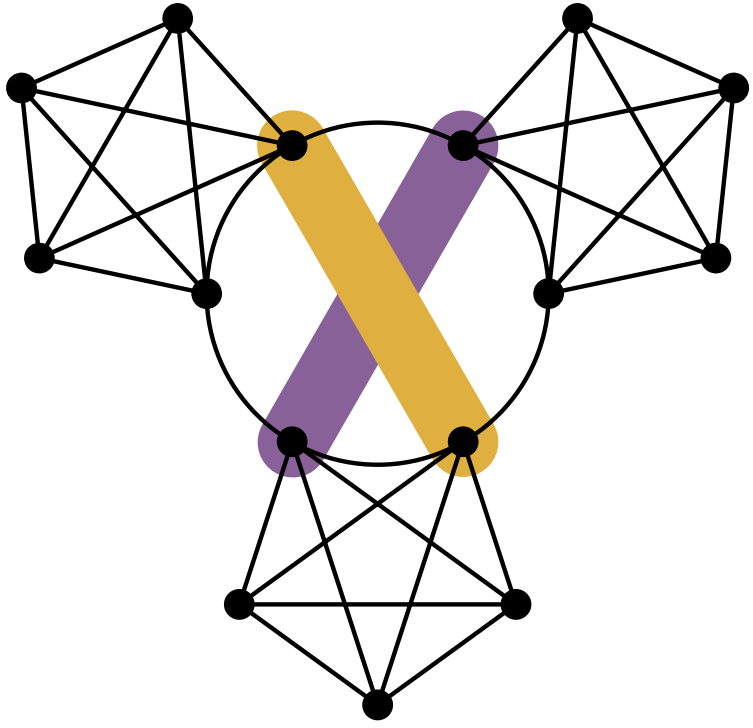
Let S be a set of separations of G.

S is *canonical* $:\Longleftrightarrow S = \varphi(S)\ \forall \varphi \in \mathrm{Aut}(G)$



canonical S



non-canonical S

S is *ε-canonical* $:\Longleftrightarrow S$ and $\varphi(S)$ ε-approximate each other $\forall \varphi \in \mathrm{Aut}$

$(T, \mathcal{V})$ is *ε-canonical* $:\Longleftrightarrow$ its set of separations is ε-canonical

Wishful Thinking

For every $q, k \in \mathbb{N}$ there are $q', \gamma, \varepsilon, \kappa \in \mathbb{N}$ such that the following holds.

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Main Result (Bourneuf, Carmesin, K. and Planken 26+)

For every $q, k \in \mathbb{N}$ there are $q', \gamma, \varepsilon, \kappa \in \mathbb{N}$ such that the following holds.

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Open: Find **exact** Tutte-decomposition for arbitrary connectivity.

Open: Directed graphs?

$k = 1$: Bowler, Gut, Hatzel, Kawarabayashi, Muzi, Reich 23

$k \geqslant 2$: ???

Open: Matroids?

Main Result (Bourneuf, Carmesin, K. and Planken 26+)

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Open: Canonical for graphs. Digraphs for $k \geqslant 2$. Matroids for $k \geqslant 4$.

arXiv: soon

Thank you :)

Slides: jan-kurkofka.eu